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AERONAUTICAL DIVISION

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PART II

REAC STUDY OF RPM CONTROL FOR THE SINGLE-SPOOL TURBOPROP ENGINE

HONEYWELL

Aeronautical Controls

CONFIDENTIAL

Security Information

#9

STUDY OF AUTOMATIC CONTROL SYSTEMS
FOR HELICOPTERS

This document has been reviewed in accordance with
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By direction of
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PART II

REAC STUDY OF RPM CONTROL
FOR THE SINGLE-SPOOL TURBOPROP ENGINE

31 July 1953

Minneapolis-Honeywell Report No. AD5143-TR4
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NOTICE

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FOREWORD

This interim report was prepared by the Research Department, Aeronautical Division, Minneapolis-Honeywell Regulator Company, under Navy Contract No. Nonr-929 (00), administered by the Office of Naval Research. The contract was initiated under the research project identified by Expenditure Accounts 46000 (Research Navy) and 46932 (Aircraft and Facilities, Navy). This is a contract for research involving the study of helicopter control systems from the point of view of automatic control of attitudes and power. This and succeeding interim reports will be followed by a final report.

ABSTRACT

This report contains a presentation of the results of an analog (REAC) computer study of the automatic control of rotor rpm of a single rotor helicopter, restricted to vertical motion and powered by a single-spool turbo-prop engine. The equations involved in the REAC simulations are included, as well as extended discussion of the simulation and associated research. Results presented include actual REAC runs, and a tabulation showing a comparison of manual control of rotor rpm with the performance of various automatic controls governing rotor rpm. The study also includes consideration of the influence of various automatic rpm control configurations on vertical maneuverability of the helicopter.

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SECTION I
INTRODUCTION

There are two types of controls which are generally considered desirable to assist the pilot in flying a helicopter. These are the autopilot, which functions to hold altitude, attitude and heading, or any one of the three; and the rpm control which maintains relatively constant rotor rpm. Many devices of both types have been developed during the past decade. (See References 1-12). Because decreases in rpm may result in dangerous coning of the rotor blades, increases may lead to excessive rotor stresses, and thrust changes produced by rpm changes may be sufficient to cause the helicopter to change altitude before recovery is possible, essentially constant rpm operation is required from a safety standpoint. Therefore, it has been deemed satisfactory for helicopter autopilots to be designed on the basis of non-varying rpm during transients. The inclusion of variation of the thrust due to changes in rpm as a variable in stability considerations is a basis for the study of which this report is a part.

Thus, the purpose of the present research is to promote the study of helicopter stability as influenced by controlled variations in rpm; and in particular, to establish criteria for autopilots and rpm controls which are integrated in such a manner that optimum rpm exists during transient changes in attitude. Besides this unique feature of this study relative to previous investigations, an additional feature is the inclusion of the pilot's manual inputs to the controls, as will be discussed later in the body of the report.

The complete study envisions the analysis of single, tandem, and co-axial rotor helicopters operating in conjunction with reciprocating (with or without supercharger), single- or double-spool turbine, ram, or pulse-jet engines and various automatic control configurations. The essential features of each portion of the study involve choosing a helicopter and engine configuration, deriving the equations of motion from basic aerodynamics and mechanics, and solving these simultaneously along with the various control system equations on the REAC and on paper by frequency or by root locus techniques.

The above procedure was followed for the case of a single-rotor helicopter powered by a single-spool turbo-prop engine, with which the present report is concerned. It was found (Reference 18) that for the hovering case, coupling of the variable rotor rpm in the attitude equations of motion of the helicopter appears to be of a secondary nature; and it seems likely that it may be eliminated from stability considerations in this flight regime. This assumption made possible the separate study

of autopilot and rpm controls for the hovering helicopter and simplified the initial phase of the study to a great extent. Because at the time the work covered in this report was initiated, the equations of motion for the hovering helicopter did not include provisions for pitch and roll but did allow for vertical motion, it was decided that the rpm control study should precede that of the autopilot. The present report is substantially a review of the work that was accomplished in simulating helicopter, engine, and a variety of controls on the REAC; and the results obtained.

SECTION II
DISCUSSION

Components of Simulation

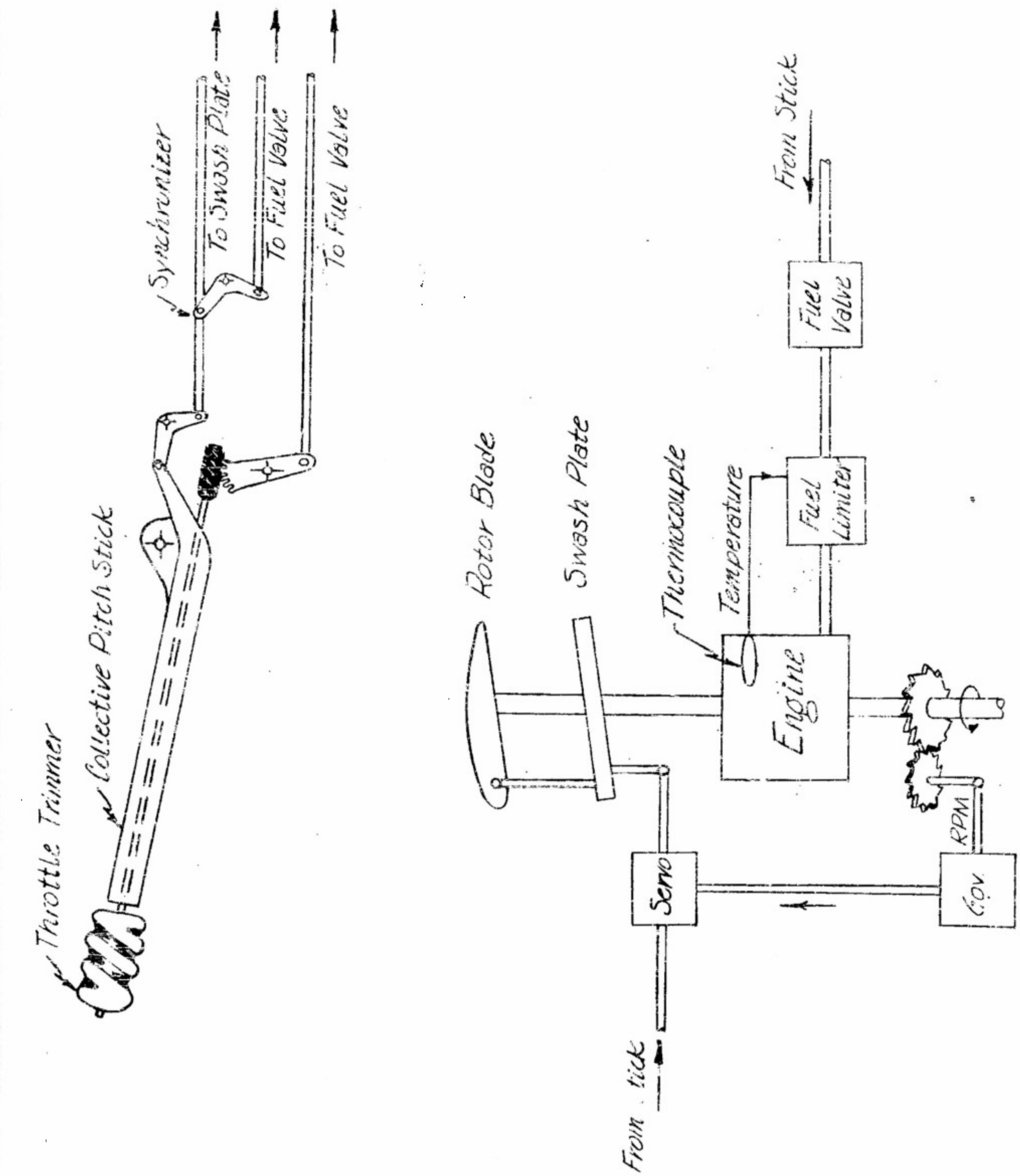
Helicopter - As stated in the "INTRODUCTION" the helicopter chosen for the present research study has a single rotor and is restricted to vertical motion. Its dynamics have been obtained from basic aerodynamical considerations. Although no actual existing helicopter may have equation coefficients identical to those used here, these are representative of those of an average or composite helicopter.

Engine - The engine chosen as the power plant for this helicopter was the Continental "Artouste I" which is a single-spool turbine engine of approximately 280 HP. The choice of turbine engine was made because at the inception of the study program the Navy felt that the use of turbine engines in helicopters would soon become of important concern. The "Artouste I" was selected primarily because when the decision was made to concentrate on turbine engines, this was the only engine for which necessary data were on hand.

The revised estimate of the present overall importance of the reciprocating engine and a consequent redirection of effort later led to temporary shelving of the turbine engine study. This prevented the report of a complete control analysis here.

Controls - In previous investigations of the automatic control of rotor rpm, little emphasis was placed on the effect of the rpm control on the vertical maneuverability of the helicopter. In the present study this consideration was deemed fundamental. It was necessary to study the performance of various automatic rpm control configurations in comparison with pilot regulation of rotor rpm, the latter serving as the standard of comparison. To satisfy the above requirements, a means of incorporating manual control of altitude and rpm was created in the form of an auxiliary device, referred to as "Steady Eddie" and described in Reference 19. By means of Steady Eddie it was possible to operate the simulated helicopter on manual pilot commands alone; or to place in operation the automatic rpm controls with altitude controlled by the pilot. The manual controls (Figure 1) are quite standard, consisting of a collective pitch stick with synchronized linkage to throttle* and an additional throttle trimmer involving a motorcycle-type twist grip on the end of the stick. The automatic features (also indicated

*The synchronization was accomplished in the REAC simulation and is further discussed on Page 16.



AERONAUTICAL DIVISION MINNEAPOLIS - HONEYWELL REGULATOR COMPANY	TITLE: Schematic of Simulated System	DEPARTMENT: Research
DOCUMENT: AD5143-TR4	CLASSIFICATION: Confidential	DRAWN BY:
		FIGURE: 1
		DATE: 7/31/53
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in Figure 1) are the rpm control, the temperature control, which for reasons to be discussed later is not shown, and the temperature limiting device. The latter limits the fuel input which can be introduced during the pilot's manual control of altitude when the turbine inlet temperature exceeds the allowable limits during automatic rpm control operation. The rpm control consists of a governor which senses rpm and a servo which drives collective pitch. The temperature control involves a thermocouple which senses temperature and sends a signal through an amplifier and compensating lead network to the fuel valve. The thermocouple also serves as a sensing device for the temperature limiter which feeds a signal to the fuel valve.

Dynamical Equations

The first step in solving any dynamics problem is, of course, the setting up of the dynamic equations expressing the motion of the components involved. The components in this case are helicopter, engine and control.

Helicopter - The derivation of the equations of motion for a hovering helicopter restricted to vertical motion is given in Reference 13. In APPENDIX B the general equations are adapted to the particular helicopter presently under discussion. The equations in their final form describe rotor torque, blade flapping and helicopter vertical motion, in that order:

$$\frac{Q_\Delta}{N} = .915\,w + 13.15\,\dot{\beta} + 30.2\,\dot{\theta}_\Delta + .58\,\Omega_\Delta - .915\,\dot{z}_c \qquad (1)$$

$$0 = -284.1\,w - 3411.7\,\dot{\beta} + 2132\,\theta_\Delta + 40.9\,\Omega_\Delta + 284.1\,\dot{z}_c - 287382\,\beta - 224.6\,\ddot{\beta} \qquad (2)$$

$$0 = 79.9\,w + 852.3\,\dot{\beta} - 532\,\theta_\Delta - 13.5\,\Omega_\Delta - 79.9\,\dot{z}_c - 97.1\,\ddot{z}_c \qquad (3)$$

The equations given here and throughout this report have been subjected to a Laplace transformation so that algebraic means alone may be used to handle them. All symbols are defined and their dimensions given in APPENDIX A.

Engine - After a perusal of the literature and a survey trip (see Reference 22) on the subject of turbine-propeller engine

dynamics, the equations as presented in Reference 14 were chosen for the simulation. This presentation was based on a linear engine analysis. The results compared favorably with experimental test data (see Reference 21). These equations are given in APPENDIX B along with the discussion of the "Artouste I" steady-state performance data and moment of inertia; these were used to obtain the following:

$$Q_E = \frac{1.45}{1+.15S} W_{f_\Delta} - .649\,\Omega_\Delta \qquad (4)$$

$$T_\Delta = \frac{3.87}{1+.15S} W_{f_\Delta} - 1.72\,\Omega_\Delta \qquad (5)$$

$$Q_E - \frac{Q_A}{N} = 2.22\,\dot{\Omega}_\Delta \qquad (6)$$

<u>Controls</u> - Because of the general nature of this research program, it was deemed advisable to construct as general a transfer function as possible to represent each control. Consequently a ratio of two polynomials was chosen at the outset. From considerations of the practical application of servomechanism theory, these polynomials, for the case of the rpm control, were reduced to the following:

$$\theta_\Delta = \frac{(K_1 + K_2/S + K_3 S)}{(1+.05S)(1+.06S)}\,\Omega_\Delta \qquad (7)$$

Controls having variable lags such as lagged rate have not been investigated as yet in this portion of the study. The two fixed lags of time constants, 0.05 and 0.06, represent the time lags of the rpm sensing device and the collective pitch servo, respectively. These values for the component lags were chosen on the basis of general experience at Minneapolis-Honeywell and it is believed that they are representative of recently developed, high-performance design. Similar considerations were involved in the reduction of the general temperature control equation to the following:

$$W_{f_\Delta} = (K_4 + K_5/S + K_6 S)\,T_\Delta \qquad (8)$$

Again variable lags in the denominator were not investigated and those fixed lags which might have existed were neglected through the following reasoning: although the sensing thermocouple has a first order lag of approximately one second, many types of compensating networks (see Reference 15) have been devised which effectively eliminate the lag; it thus seemed unnecessary to include any lag for this portion of the controls. Since solenoid

operated fuel valves can be designed to have practically no dynamics the servo lag was also neglected. The temperature limiter control is represented only by a gain because it is an on-off proportional device which uses the same thermocouple and servo as the temperature control and consequently has no appreciable lags.

Block Diagram

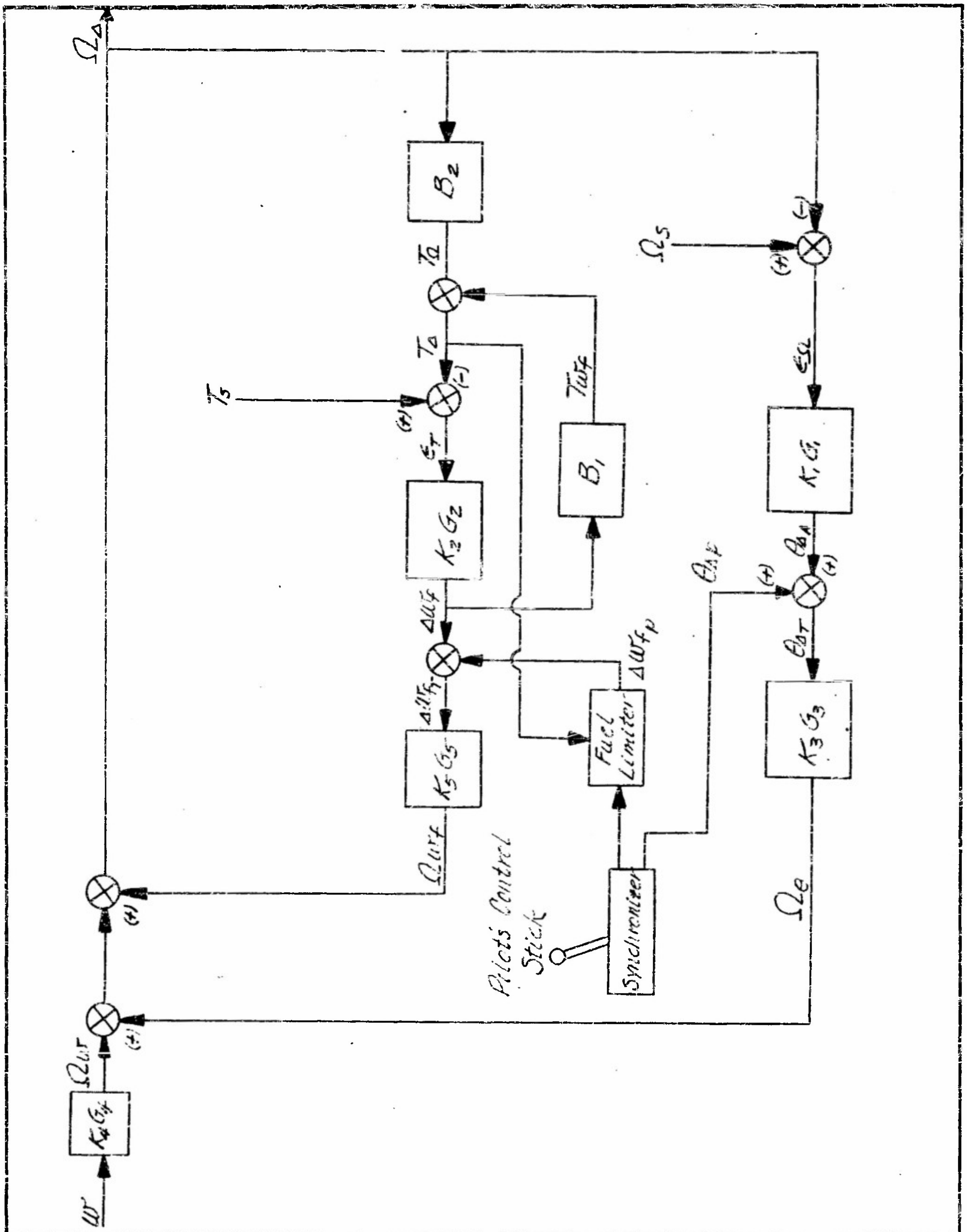
The complete system is represented in block diagram form in Figure 2. The input to the system is vertical gust and the output is rpm. The transfer functions K_1G_1 and K_2G_2 represent the rpm and temperature controls, respectively; and the other transfer functions (B_1, B_2, K_3G_3, K_4G_4, K_5G_5) representing helicopter engine interrelationships will be found in Reference 16, which is a root locus analysis of a similar system.

In the particular system discussed here, rpm is controlled by automatically varying collective pitch, while in Reference 16 rpm is controlled by varying fuel flow. The trasfer functions are not directly involved in this study and will not be referred to again.

REAC Simulation

Setup – The photographs, Figures 3a, 3b, and 3c show the analog computing facility which was utilized in conducting the present study. The simulation includes helicopter, engine, rpm control, turbine inlet temperature control, temperature limiter, and "Steady Eddie". The latter is a simulated helicopter cockpit designed to present a high degree of reality to the problem for a pilot and to simplify greatly the tasks of the REAC operator (see Reference 19). It consists of an operator's seat on which are located a synchronized collective pitch and throttle lever, a throttle trimmer, and a control disengage button; and a cabinet which contains rpm, altitude and turbine temperature meters for the pilot, control gain pots, a switch to turn on several recorders simultaneously, a computer's operate switch, a step function switch, REAC overload lights, an excessive turbine temperature warning light, and buttons making possible the engaging of rpm control by the REAC operator. (See Figure 3c). The remainder of the simulation is on the REAC and follows standard practice. Auxiliary equipment includes two four-channel Sanborn recorders and a gust generator. The latter consists of a moving strip of film and a photoelectric cell. This combination provided sharp inputs to the system which were shaped by a lag network to simulate vertical gusts.

Final Equations The final equations solved on the REAC are, with the exception of control equations 7a and 8a, derived in APPENDIX B, and are given on the following page:



AERONAUTICAL DIVISION MINNEAPOLIS - HONEYWELL REGULATOR COMPANY	TITLE: Block Diagram of Complete System	DEPARTMENT: Research	
		DRAWN BY:	FIGURE: 2
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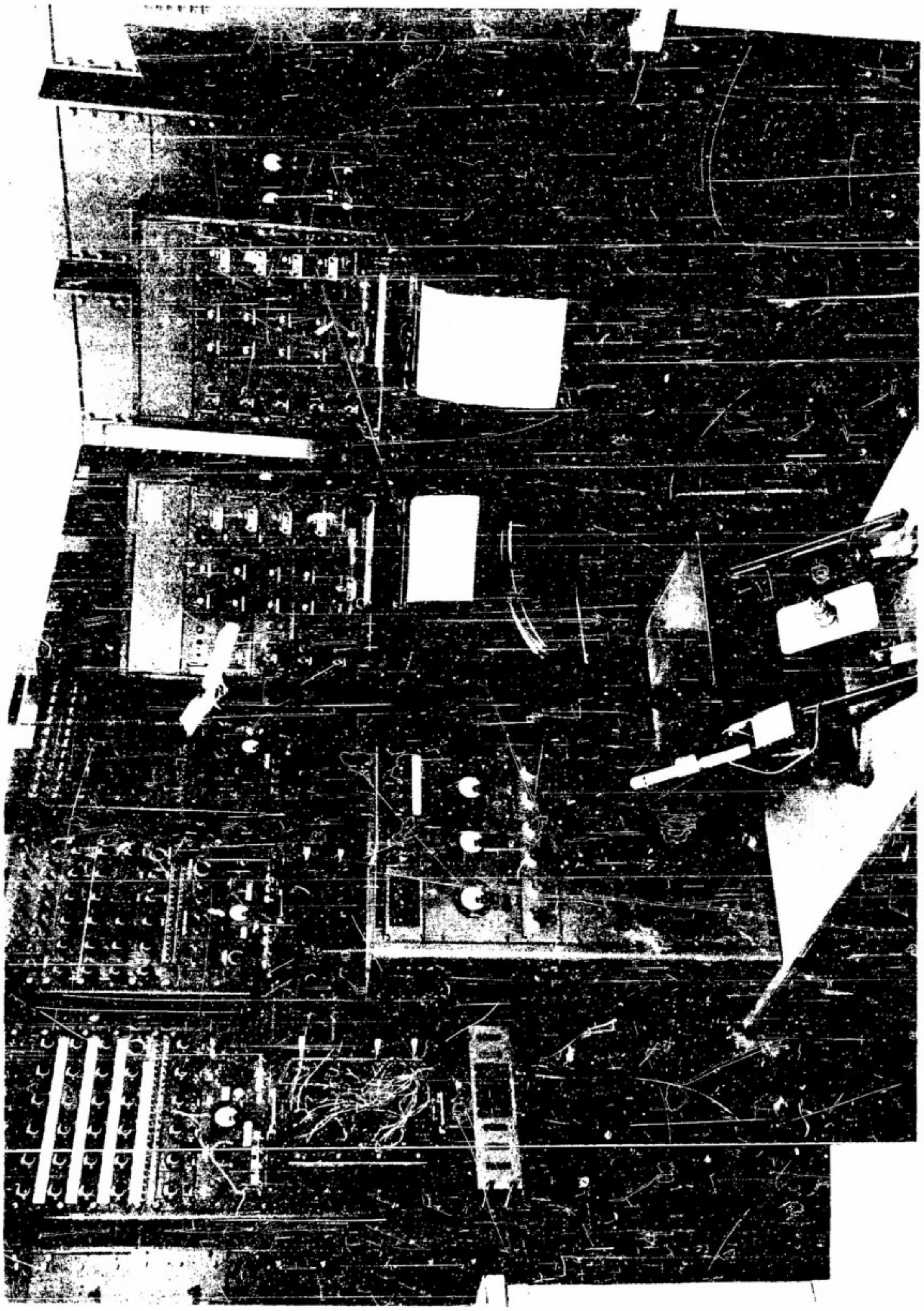
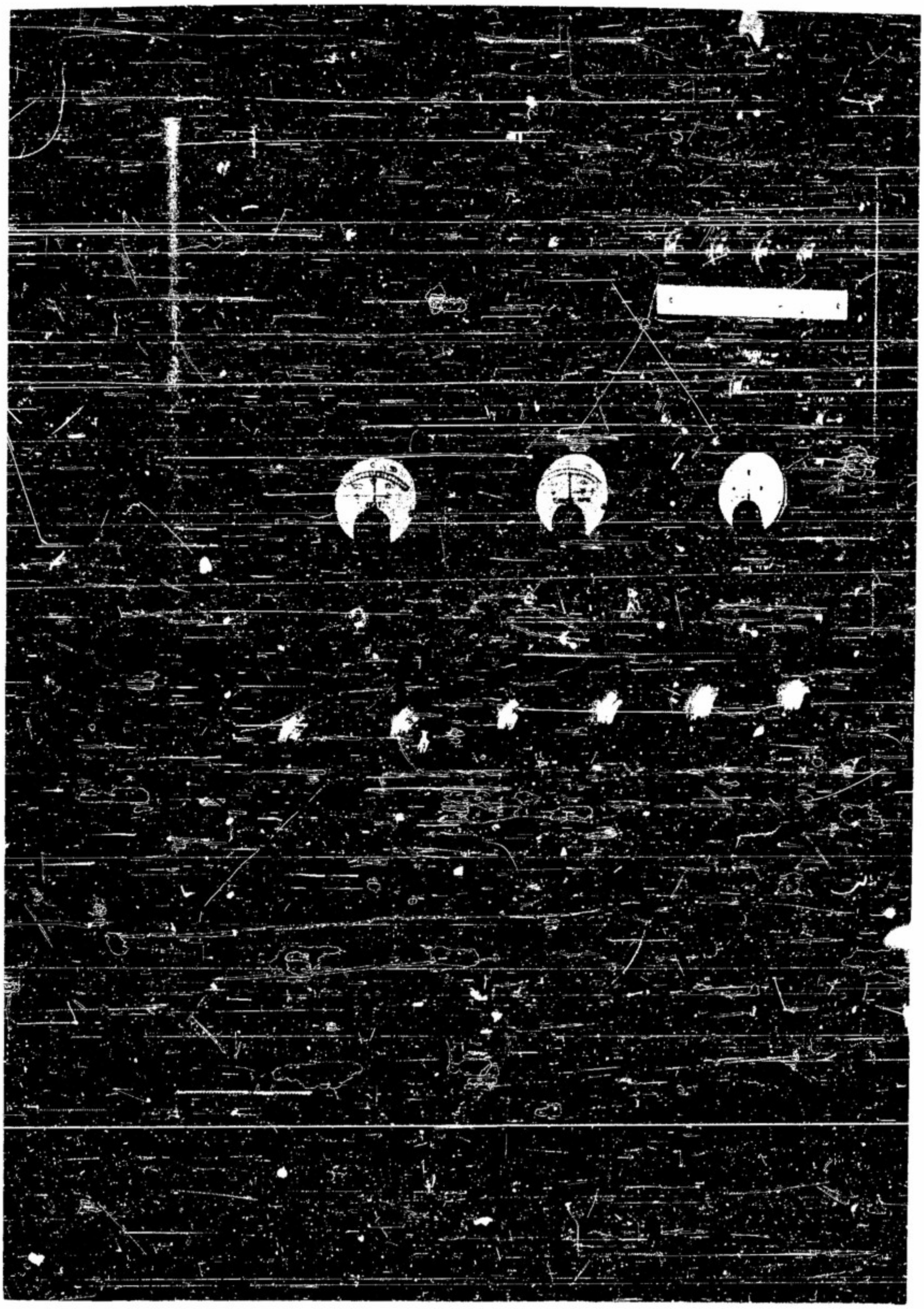


AERONAUTICAL DIVISION MINNEAPOLIS-HONEYWELL REGULATOR COMPANY		TITLE: REAC and Steady Eddie (Cyclic Pitch Stick and VGI Display Not Connected)	DEPARTMENT: Research
DOCUMENT: AD5143-TR4	CLASSIFICATION: Confidential		DRAWN BY: / FIGURE: 3a
			DATE: 7/31/53 / PAGE NO: - 9 -



AERONAUTICAL DIVISION MINNEAPOLIS - HONEYWELL REGULATOR COMPANY	TITLE:	DEPARTMENT: Research
DOCUMENT: AD5143-TR-4	CLASSIFICATION: Confidential	Steady Eddie Control and Display Panel	DRAWN BY:	FIGURE: 3b
			DATE: 7/31/53	PAGE NO: - 10 -

AERONAUTICAL DIVISION MINNEAPOLIS - HONEYWELL REGULATOR COMPANY	TITLE: REAC and Steady Eddie in Operation	DEPARTMENT: Research	
		DRAWN BY:	FIGURE: 3c
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$$\ddot{\Omega}_\Delta = 4.35 W_{f_\Delta} - 3.69\Omega_\Delta - 2.75 w - 39.4\dot{\beta} - 90.1\theta_\Delta + 2.75\dot{\jmath}_c$$
$$- 7.2\dot{\Omega}_\Delta - .411\dot{w} - 5.91\ddot{\beta} - 13.5\dot{\theta}_\Delta + .411\ddot{\jmath}_c \qquad (9)$$

$$\ddot{\beta} = -1.265 w - 15.18\dot{\beta} + 9.5\theta_\Delta + .182\Omega_\Delta + 1.265\dot{\jmath}_c - 1278\beta \qquad (2a)$$

$$\ddot{\jmath}_c = .823 w + 8.77\dot{\beta} - 5.48\theta_\Delta - .139\Omega_\Delta - .823\dot{\jmath}_c + .631\ddot{\beta} \qquad (3a)$$

$$\dot{T}_\Delta = 25.8 W_{f_\Delta} - 11.48\Omega_\Delta - 6.67 T_\Delta - 1.72\dot{\Omega}_\Delta \qquad (5a)$$

$$\ddot{\theta}_\Delta = 333 K_1 \Omega_\Delta + 333 K_2 \Omega_\Delta / s + 333 K_3 \dot{\Omega}_\Delta - 333\theta_\Delta$$
$$- 36.7\dot{\theta}_\Delta \qquad (7a)$$

$$W_{f_\Delta} = -K_4 T_\Delta - K_5 T_\Delta / s - K_6 T_\Delta \qquad (8a)$$

The portion of the REAC diagram (Figures 4a, 4b and 4c) devoted to each of the above equations is noted on that diagram.

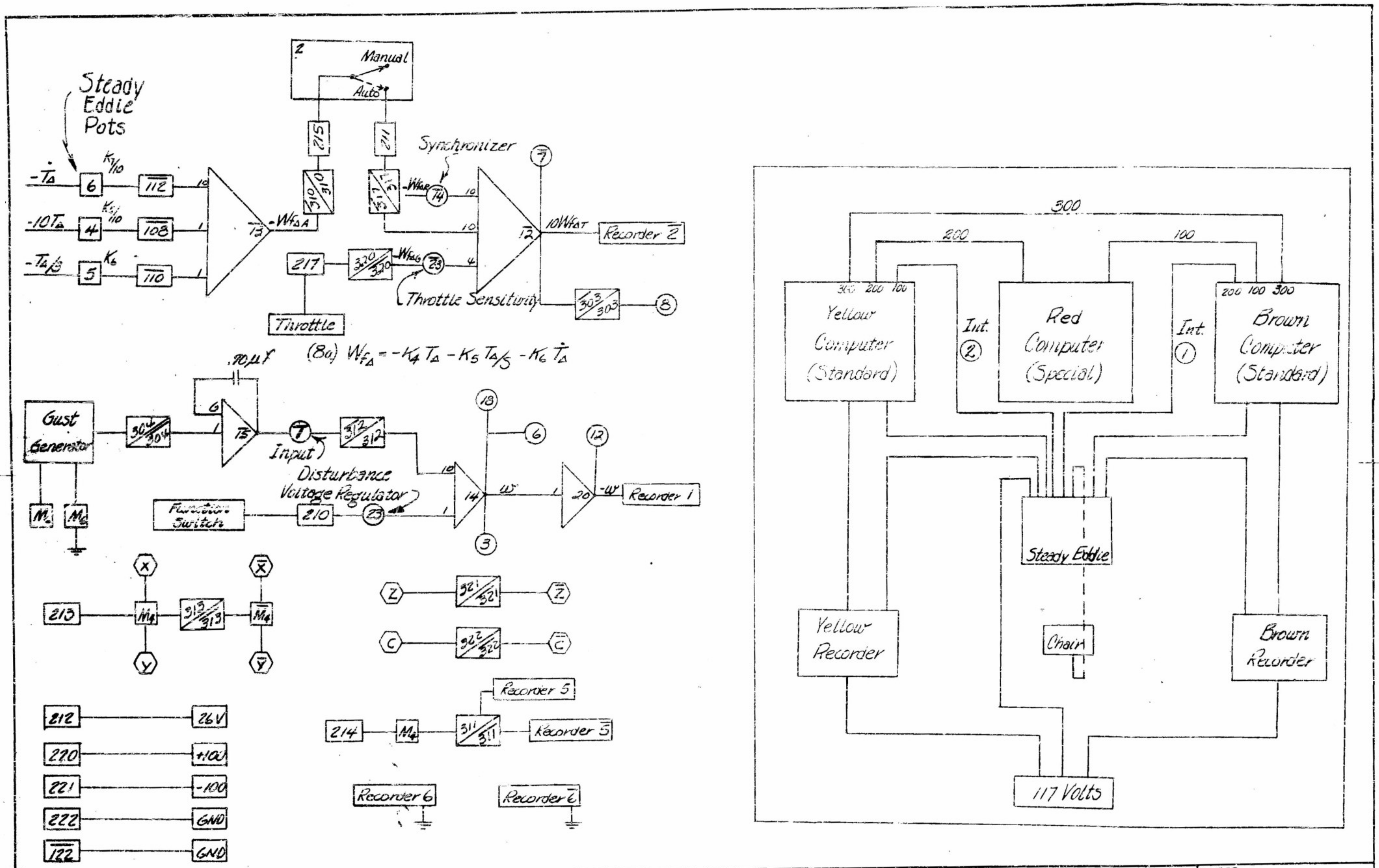
REAC Diagram

The general layout of the equipment involved in the simulation and the interconnects are shown on the right side of Figure 4a. Referring now to the main portion of the REAC diagram, amplifiers and pots located on the yellow computer are indicated by a bar over their numbers; those located on the red computer have bars under their numbers; and those on the brown computer have unmarked numbers. Recorder channels are similarly marked. Square pots are located on "Steady Eddie".

Special Features

A description of important features of the REAC simulation follows:

Temperature Limiter - This on-off control is shown in Figure 4b. A relay amplifier R controls the two relays (found at the ends of the lines leading from the amplifier) according to signals received from amplifiers 6 and interconnect 201. The latter is the turbine temperature signal and the former is a selected maximum limit to be imposed on temperature. When the temperature actually exceeds the set maximum allowable value the relay operation begins, simultaneously disconnecting the pilot's fuel input (a deviation from steady state) and connecting -100 volts through a pot to



AERONAUTICAL DIVISION MINNEAPOLIS - HONEYWELL REGULATOR COMPANY	TITLE: REAC DIAGRAM	DEPARTMENT: Research
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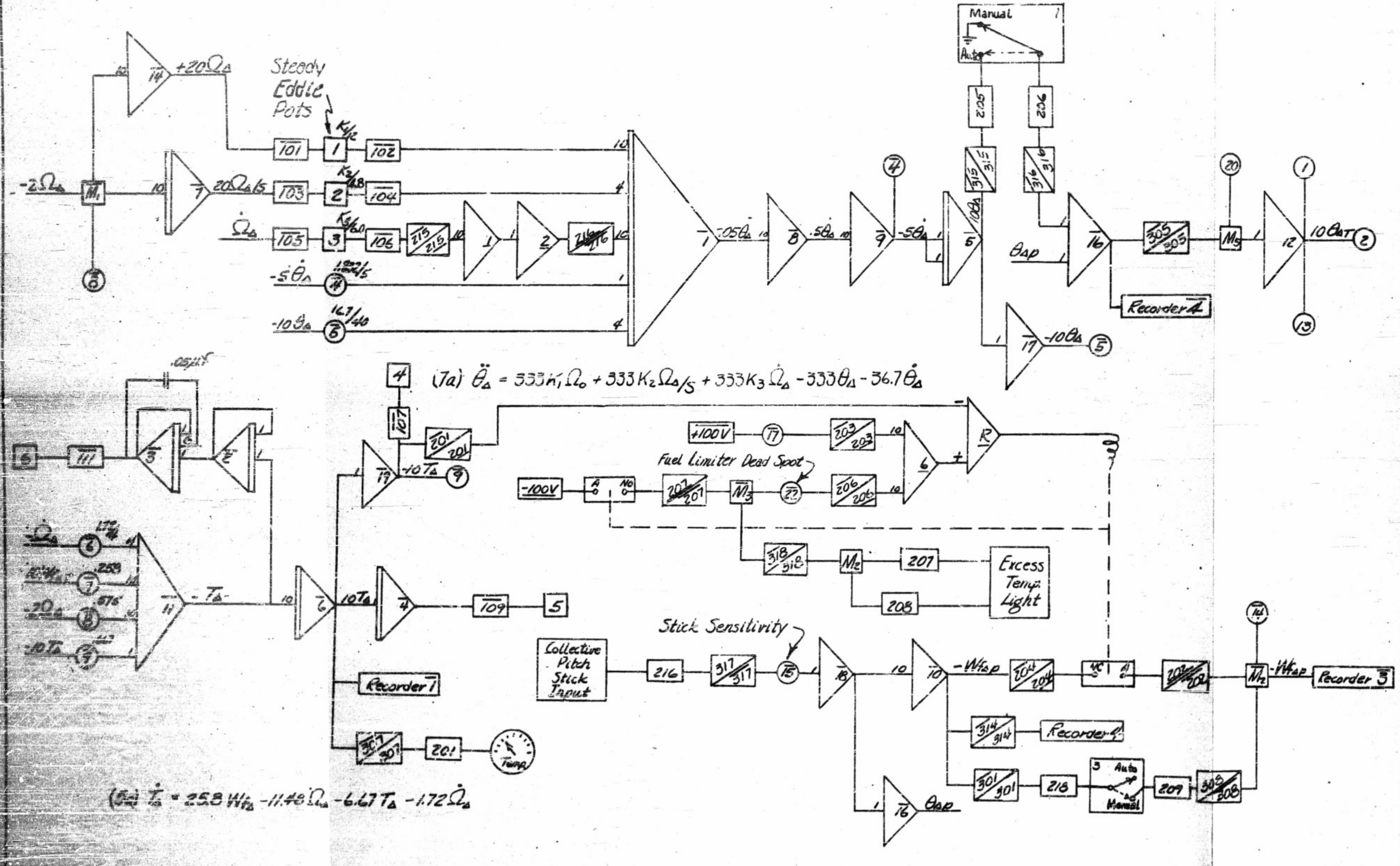

Aeronautical Division, Minneapolis-Honeywell Regulator Company	TITLE: REAC DIAGRAM	DEPARTMENT: Research
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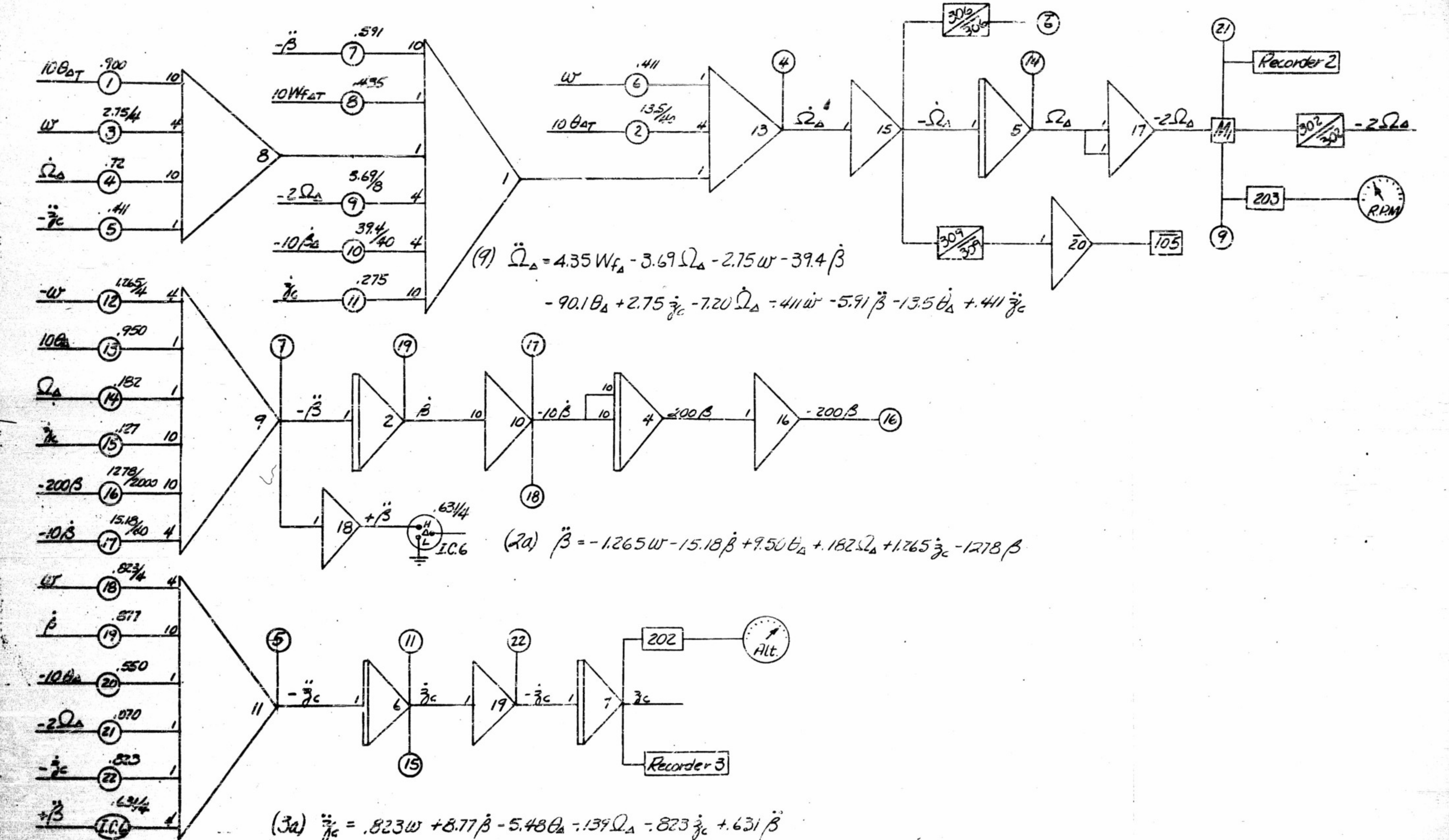

(9) $\ddot{\Omega}_{\Delta} = 4.35 W_{f_{\Delta}} - 3.69\Omega_{\Delta} - 2.75 w - 39.4\dot{\beta} - 90.1\theta_{\Delta} + 2.75\dot{\bar{z}}_c - 7.20\dot{\Omega}_{\Delta} - .411\dot{w} - 5.91\ddot{\beta} - 13.5\dot{\theta}_{\Delta} + .411\ddot{\bar{z}}_c$

(2a) $\ddot{\beta} = -1.265 w - 15.18\dot{\beta} + 9.50\theta_{\Delta} + .182\Omega_{\Delta} + 1.265\dot{\bar{z}}_c - 1278\beta$

(3a) $\ddot{\bar{z}}_c = .823 w + 8.77\dot{\beta} - 5.48\theta_{\Delta} - .139\Omega_{\Delta} - .823\dot{\bar{z}}_c + .631\ddot{\beta}$

AERONAUTICAL DIVISION MINNEAPOLIS-HONEYWELL REGULATOR COMPANY	TITLE: REAC DIAGRAM	DEPARTMENT: Research	
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		DATE: 7/31/53	PAGE NO: - 15 -

amplifier 6, thereby lowering the set voltage level on the + side of the relay amplifier. As temperature begins to return to its steady-state value(a consequence of the elimination of the extra fuel input) the relay amplifier will not reverse its operation at the original voltage (corresponding to maximum allowable temperature) but will allow temperature to drop slightly below this level before action begins. This gives a dead-spot effect controlled in magnitude by pot 22. Pot 17 controls the original or set maximum allowable voltage level which is determined by the level on the engine steady-state performance curves at which steady-state operation is occuring. As soon as relay operation is reversed, the fuel input corresponding to the position of the pilot's collective pitch stick returns to its former value. The operation is repeated at the frequency desired. This frequency depends on how high the average temperature should be to give minimum acceptable power output of the engine.

Auto-Manual Switches - Three switches of this type are shown. Switch Number 1 effectively breaks the automatic control input to collective pitch when on manual, as can be seen in Figure 4b, in the simulation of equation 7a. Switch Number 3 on the same figure is an auxiliary to the temperature limiter simulation and is used to complete the pilot's fuel input circuit when on manual operation even when the temperature limiter is operating. In manual, excessive temperature is indicated by the warning light shown in the temperature limiter circuit, but no automatic limiting exists. The responsibility here rests soley with the pilot. The purpose of Switch Number 2 on Figure 4a is to break the temperature feedback shown in the simulation of equation 8a.

Pilot's Signal Inputs - The pilot's collective pitch and synchronized fuel input signals through a collective pitch stick shown in Figure 4b were fed into the system at a ratio determined by the relative effectiveness of a unit change in fuel flow and a unit change in collective pitch on rpm as given by the relationship between these variables in Equation 9 shown simulated on Figure 4c. Manual fuel input from collective pitch stick movement enters amplifier 12 (equation 8a), Figure 4a, as does the fuel input from the twist grip throttle. Collective pitch enters amplifier 16 (equation 7a), Figure 4b.

Gust Inputs - The gust disturbance regulator shown in Figure 4a is designed to give the REAC operator a variety of gust input shapes. The function switch is used for step inputs, while the gust generator is used for random inputs which maybe shaped by varying the condensor in the lag network in Amplifier 15.

Research Program

By continual changing of control gains it was possible to obtain a range of suitable controls varying in complexity from a simple, proportional control on rpm alone to a combination of proportional + integral + rate on the rpm control and a proportional + integral + rate on the temperature control.

1. Purpose of Temperature Control Most of the automatic control configurations included in this investigation did not include feedback control of turbine temperature. It was not felt that the controlling of turbine inlet temperature to a steady-state value was a primary requirement inherent in this study of the turbo-prop engine. However, an awareness of the effect of changes in fuel (to prevent large changes in the temperature of the engine) on the response of the helicopter to called-for power increases is certainly a factor of major concern and was handled in the following manner. The temperature control involved in this study was considered to be a research tool rather than a full-fledged control, and was used to show the relative effect of temperature feedback to the fuel valve on thrust response. This control constitutes a supplement to the temperature limiter and limits both + and - deviations. Proportional feedback alone was found to be sufficient to satisfy the control requirements. The addition of integral tended to return the temperature to its set value. This imposed too great a limitation on temperature which should be allowed to hold at maximum for some definite value of time - one minute in this case. (Reference 17). The addition of rate feedback speeded up the temperature response; but the decreased response time is valueless in this instance as will be seen on the REAC runs. Consequently, the final control feedbacks are proportional, integral and rate for the rpm control; and proportional alone for the research tool called the temperature control.

2. Purpose of Temperature Limiter Somewhat further discussion may be advisable concerning the temperature limiter. As stated in the previous section, the effect of this limiter is to limit the pilot's fuel input (to control altitude) when exceeding maximum allowable temperature while on automatic rpm control. This is accomplished through an on-off relay action, the rapid oscillatory effect of which will not show up on the REAC runs photographs; therefore, lines of maximum temperature and maximum and average fuel flow for the periods during temperature limiter operation are shown instead since these are the actual values of interest. Like temperature control, the temperature limiter is somewhat artificial. An actual temperature limiter would be designed to control fuel and/or load according to the error between actual temperature and maximum allowable. Because the added complication in simulating such a system is unwarrented in this study, and since a similar effect can be produced by the means adopted, the simple on-off device was used. This has one undesirable feature. A tight rpm control picks up the fluctuations in

rpm caused by fluctuations in fuel (which are due to the operation of the limiter) and feeds them to the collective pitch servo with the intention of damping them out by rapid variations in collective pitch. These collective pitch oscillations with an approximate magnitude of less than 0.25° would not exist in a normal temperature limiting scheme, and would probably not exist in the present study if rotor blade inertia around a blade span axis were considered. These collective pitch oscillations are replaced by an average collective pitch line representing normal operation because, as in the case of fuel flow and temperature, these light markings of the recorder pen will not photograph and are meaningless in any event.

3. Recorded Variables As has been indicated in APPENDIX B, the variables discussed in this report are actually deviations from a set operating point and consequently actual magnitudes are relatively small. Those variables which are of primary interest and are recorded are collective pitch (θ_Δ), fuel flow ($W_{f\Delta}$), turbine inlet temperature (T_Δ), rpm (Ω_Δ), altitude (z_c), gust input (w), and pilot's stick input. One additional recording channel was utilized in recording the temperature limiter relay action.

4. Control Criteria Only those controls that satisfied certain prescribed requirements were considered acceptable. These criteria were as follows:

a. RPM : Deviations of as much as 25 out of 6000 were considered tolerable. This decision was based on the observation that the deviations during manual operation were of this order of magnitude and that the control should be expected to do as well as the pilot, but not necessarily any better. Also rapid oscillations were considered unacceptable.

b. Turbine Inlet Temperature: No maximum negative inlet temperature increment was set. The maximum positive increment as indicated in APPENDIX B was somewhat arbitrary. The actual value depends upon the set operating point on the engine performance curves. For the major portion of this study the operating point was chosen to lie 100°F below the maximum allowable temperature line on the 6000 rpm line.

c. Altitude: Variations in altitude were expected to be held to a minimum, with the operator experiencing satisfactory response to vertical maneuver commands.

d. Fuel Flow: Only the pilot's stick fuel input required limiting and this was done by allowing 100 pounds/hr of fuel flow for full stick travel as indicated in APPENDIX B.

e. Collective Pitch Variations in collective pitch were required to be smooth. Maximum stick travel gives a 5° pitch range. This range is small compared with that of an actual collective pitch stick; but it is satisfactory for present purposes.

5. Typical Run Before entering into a discussion of the REAC recordings which constitute the results of the present study it may be worthwhile to describe a typical run. This is shown as a step-by-step process below.

A. Basic Run

1. Set values of rpm control pots.
2. Turn on recorders.
3. Flip REAC switch to operate.
4. Put in step up-gust with function switch.
5. Hold altitude to minimum departure from steady-state setting by manually moving collective pitch stick (which introduces synchronized collective pitch and fuel valve signals to the simulation).
6. Put in step down-gust with function switch.
7. Hold altitude to minimum departure from steady-state by moving collective pitch stick.
8. Give full stick in either direction.
9. When altitude has departed 25 feet from steady-state value, give full stick in opposite direction. Hold until 25 feet in opposite direction is reached and then reverse the stick once more.
10. Turn off system and reset control pots for next run.

B. Variations in Basic Run

The following variations were made in the basic run.

1. The temperature limiter setting was changed, effectively changing the engine operating point for a series of runs.
2. Step gust input was replaced by gust generator for another series of runs.
3. The setting of temperature control pots was additional in some runs.
4. Automatic control was switched off and the system was operated in manual.

SECTION III
RESULTS

The results of the present REAC study are presented in two forms. First, a table is presented (Figure 5) which shows the gain-pot settings and gains for the several runs included here and a comparison between the runs from several basic considerations. Second, the runs themselves are shown for the purpose of further run comparison according to the interests of the reader and of having a permanent record of those runs which are of special interest.

Tabulation of Runs

The discussion of the REAC runs will be centered around the table mentioned above. The numbered columns give the following information:

Column 1	Run Number
Column 2	Pot one, proportional rpm control gain pot setting
Column 3	Pot two, integral rpm control gain pot setting
Column 4	Pot three, rate rpm control gain pot setting
Column 5	Pot four, proportional temperature control gain pot setting
Column 6	Pot seventeen, maximum temperature level setting
Column 7	A, departure of engine rpm from set value after down-gust
Column 8	B, departure of altitude in feet after down-gust
Column 9	C, time in seconds for helicopter to climb 25 feet after rapid full stick input
Column 10	D, proportional rpm control gain
Column 11	E, integral rpm control gain
Column 12	F, rate rpm control gain
Column 13	G, proportional temperature control gain

The purpose of columns 7, 8, and 9 is to give some indication of the merits of the various automatic controls relative to each other and to manual control. Generally speaking, two types of information are given in these columns: first, the relative capabilities of each system in controlling rpm and second, the relative helicopter altitude response to stick changes for each control system.

It is readily seen that the outstanding differences between manual and a typical automatic rpm control are relatively higher rpm deviation after a step gust input and relatively more rapid response of the helicopter following stick movements in manual as compared with automatic.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Run No.	Pot 1	Pot 2	Pot 3	Pot 4	Pot 17	A, Ω_Δ	B	C	D, K_1	E, K_2	F, K_3	G, K_4
Manual		0	0	0	.50	.50	17	4.0	0	0	0	0
51	.10	0	0	0		22	8	6.4	1.2	0	0	0
11	0	.10	0	0		110	8	6.4	0	.48	0	0
5		0	1.00	0		13	11	6.4	0	0	6.0	0
31	.05	.05	0	0		53	12	6.2	.6	.24	0	0
54	.10	0	.50	0		11	7	6.4	1.2	0	3.0	0
19	0	.50	.50	0		10	10	6.5	0	2.4	3.0	0
119	1.00	.50	.50	0		2	7	6.6	1.2	2.4	3.0	0
19-1	0	.50	.50	.02		5	9	6.2	0	2.4	3.0	.2
19-2	0	.50	.50	.05		8	9	6.8	0	2.4	3.0	.5
19-3	0	.50	.50	.10		12	13	9.5	0	2.4	3.0	1.0
51-1	.10	0	0	.02		15	6	6.5	1.2	0	0	.2
51-2	.10	0	0	.05		20	10	7.1	1.2	0	0	.5
51-3	.10	0	0	.10		25	14	10.5	1.2	0	0	1.0
19-G	0	.50	.50	0	.50	-	-	-	0	2.4	3.0	0

FIGURE 5. Tabulation of Runs

In all fairness to the comparison it should be made clear that the control of rpm in manual is not a simple proposition and that a person (such as a pilot) whose movements are relatively well coordinated might be able to do a better job of holding rpm to a set value than did the REAC operator. A departure of 25 rpm during manual operation is considered to be more representative of actual pilot capabilities. From Column C it would appear that the altitude response of the helicopter in manual is decidedly faster than on automatic; but this is only true during climb when high power is required, with consequent temperature and, therefore, fuel limiting by the automatic controls. With no automatic temperature limiting the responses would be more nearly the same. They would also have been more nearly the same if the operator had backed off on fuel requirements at the instant of the lighting of the excessive temperature light when on manual. One other word of caution when interpreting results is necessary, because the step gust inputs, although always of the same magnitude, vary slightly in duration since they were put in manually through a step function switch. Therefore, some controls may have been given a slightly more vigorous test than others. In general, it is felt that the results are indicative and show definite optimums and trends.

It will be noticed that although the symbol list in APPENDIX A gives Ω_{Δ} as radians/second (dimensions used in equations) the text discusses rpm (generally used to denote rotor angular velocity). One radian/second equals approximately 10 rpm and it was therefore a simple matter to read rpm from the REAC tracing rather than radians/second, thereby being consistent with general practice.

Recordings

The runs included in the report and compared in the table were chosen to show optimums of various types of controls or effects of variations in some parameter of the system, and are discussed individually below.

1. Manual (Figure 6)

   The purpose of this run is to establish a criterion for deviations in rpm which will determine a maximum magnitude that should not be exceeded in any acceptable automatic control run.

2. Run 51 – Proportional rpm Control (Figure 7)

   A decrease in gain led to rpm deviations larger than were considered tolerable while higher gain values led to instability. Helicopter altitude response is good. RPM oscillations might be objectionable here.

3. Run 11 - Integral rpm Control (Figure 8)

   This control varied with gain settings from very unsatisfactory to unstable. RPM oscillations are extreme. Altitude response is good. This is the optimum run for its type.

4. Run 5 - Rate rpm Control (Figure 9)

This type of control is considered unsatisfactory although less so than the integral control. The effect of the lack of reset characteristics is more pronounced here than in the proportional type of control. Altitude response is fair. This is the optimum run for its type.

5. Run 31 - Proportional + Integral rpm Control * (Figure 10)

Peak rpm deviations are slightly high. The effect of reset is apparent. Increasing the integral gain causes high rpm oscillations while increase in the proportional gain tends to produce instability. Altitude response is fair.

6. Run 54 - Proportional + rate rpm Control (Figure 11)

This control does a good job. The use of rate feedback allows higher proportional gain for the same degree of stability, and thus tighter control is possible. Decreasing rate was degenerative while increases offered only slight improvement. Decreasing the proportional gain led to poorer rpm response while increasing the gain led to poorer helicopter response. Altitude response is good.

7. Run 19 - Integral + rate rpm Control (Figure 12)

This control is very interesting and should be explored further when the turbine engine study is resumed. Changes in integral gain haven't much effect; but it is felt that the range of this gain variation should be extended. Increasing rate gain improves the control only slightly while decreases lead to instability. Altitude response is fair to good.

8. Run 119 - Proportional + integral + rate rpm Control (Figure 13)

As can be seen on the REAC run or in the table this control holds rpm almost to zero deviation from steady state (in choosing between a control of this type, relatively expensive and complex but very tight, and a less expensive and complex control which is somewhat less effective, the rpm control designer must be guided by the needs of the helicopter and autopilot designer). Either up or down variations in integral control gain caused degeneration of the system. Although increases in proportional and rate control gains may give a slightly tighter rpm control it is felt that this is unnecessary. Altitude response is good.

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* The use of integral feedback is required if steady-state error in rpm is to be eliminated, but it may be that small steady-state errors would be tolerable in certain applications.

9. Runs 19- 1, 2, 3; 51-1, 2, 3 - RPM Control + varying degrees of proportional temperature Control (Figures 14-19)

These runs are included to show relative effects on helicopter response of various temperature controls. The comparison involves the rpm controls 19 and 51 and three temperature controls increasing in tightness with run number. It was generally true that the effect of the temperature control on rpm depended to a small extent on the particular rpm control involved. An unforeseen circumstance exists in that a loose control on temperature in conjunction with the temperature limiter results in smaller rpm deviation and better helicopter altitude response than that given by the configuration involving no temperature control. This is apparently due to the fact that the temperature limiter, when in operation, limits the fuel to an average value between steady state and maximum while the temperature control tends to prevent the temperature from reaching the limiting value thereby allowing more than average fuel and consequently allowing better helicopter and rpm response. Tight temperature control, of course, results in large rpm deviations and poor helicopter altitude response.

10. Run 19-G - RPM Control 19 with gust generator inputs (Figure 20)

The purpose of this run is to test the controls under somewhat more realistic gust conditions. RPM and altitude responses are comparable to those of Run 19.

A series of runs involving a lower maximum temperature deviation were made; but nothing new was determined here, so none are presented.

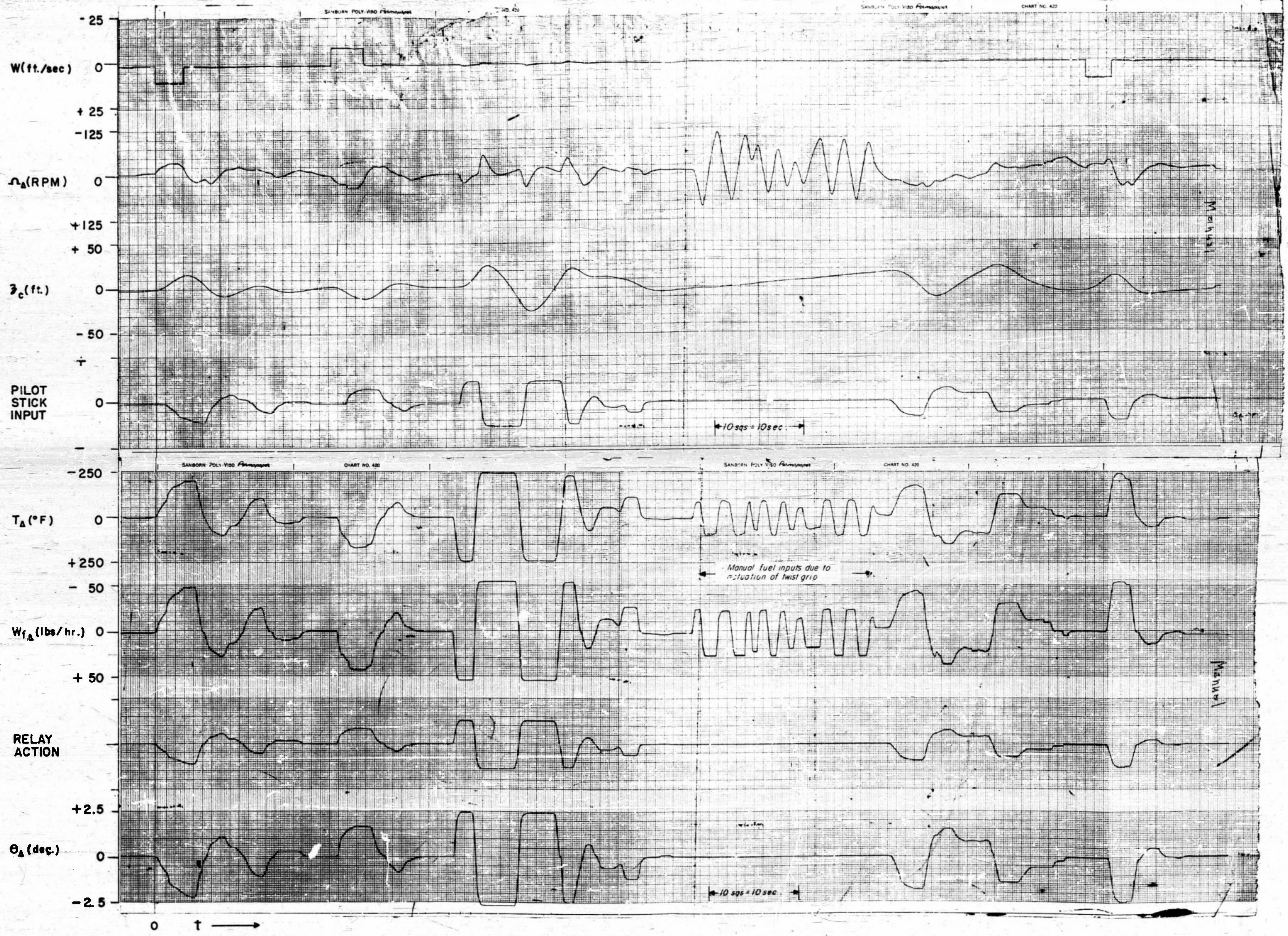

Figure 6. - Manual Run, $K_1 = 0$
$K_2 = 0$; $K_3 = 0$; $K_4 = 0$

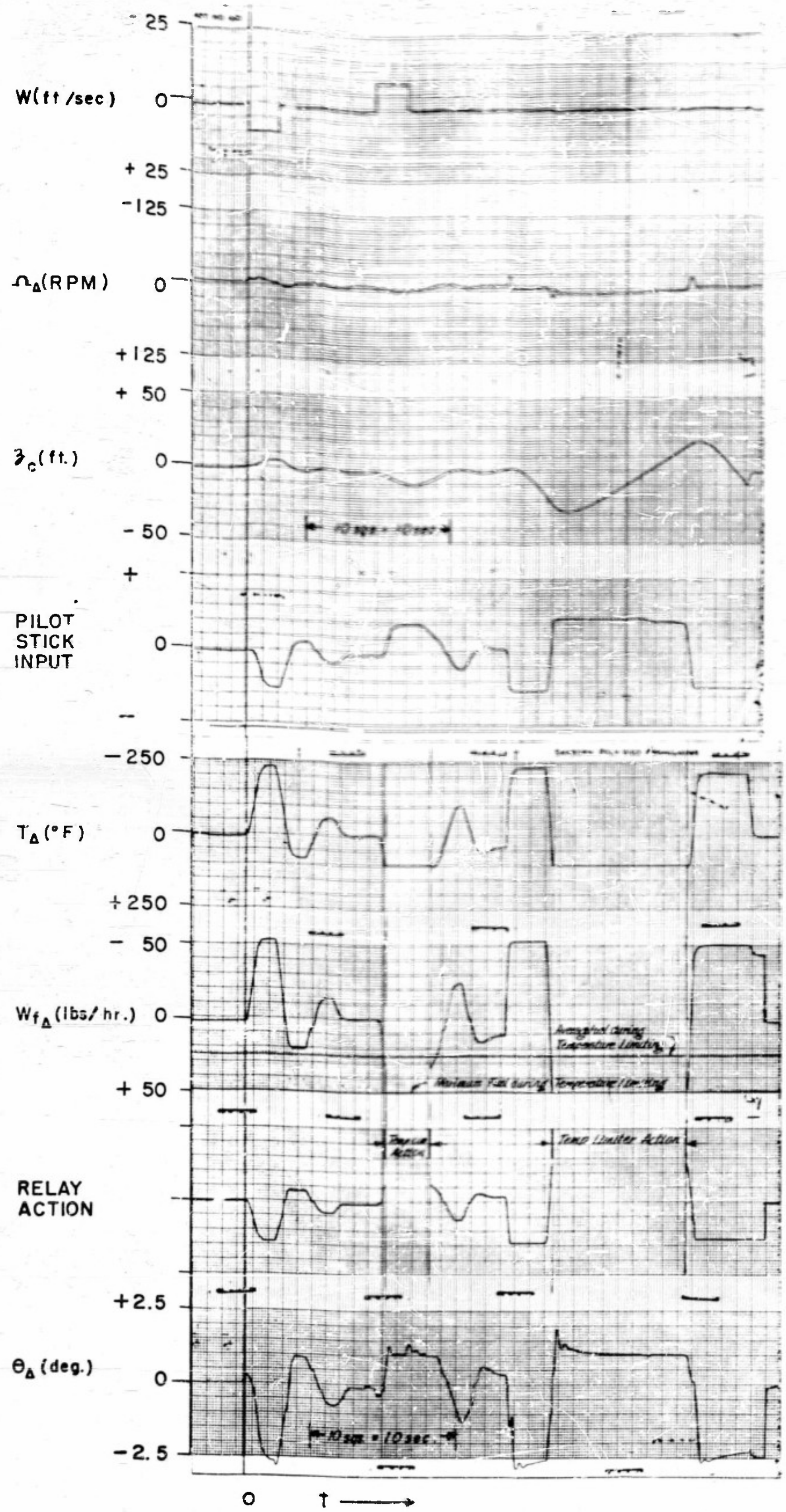

Figure 7. - Run 51, $K_1 = 1.2$; $K_2 = 0$; $K_3 = 0$; $K_4 = 0$

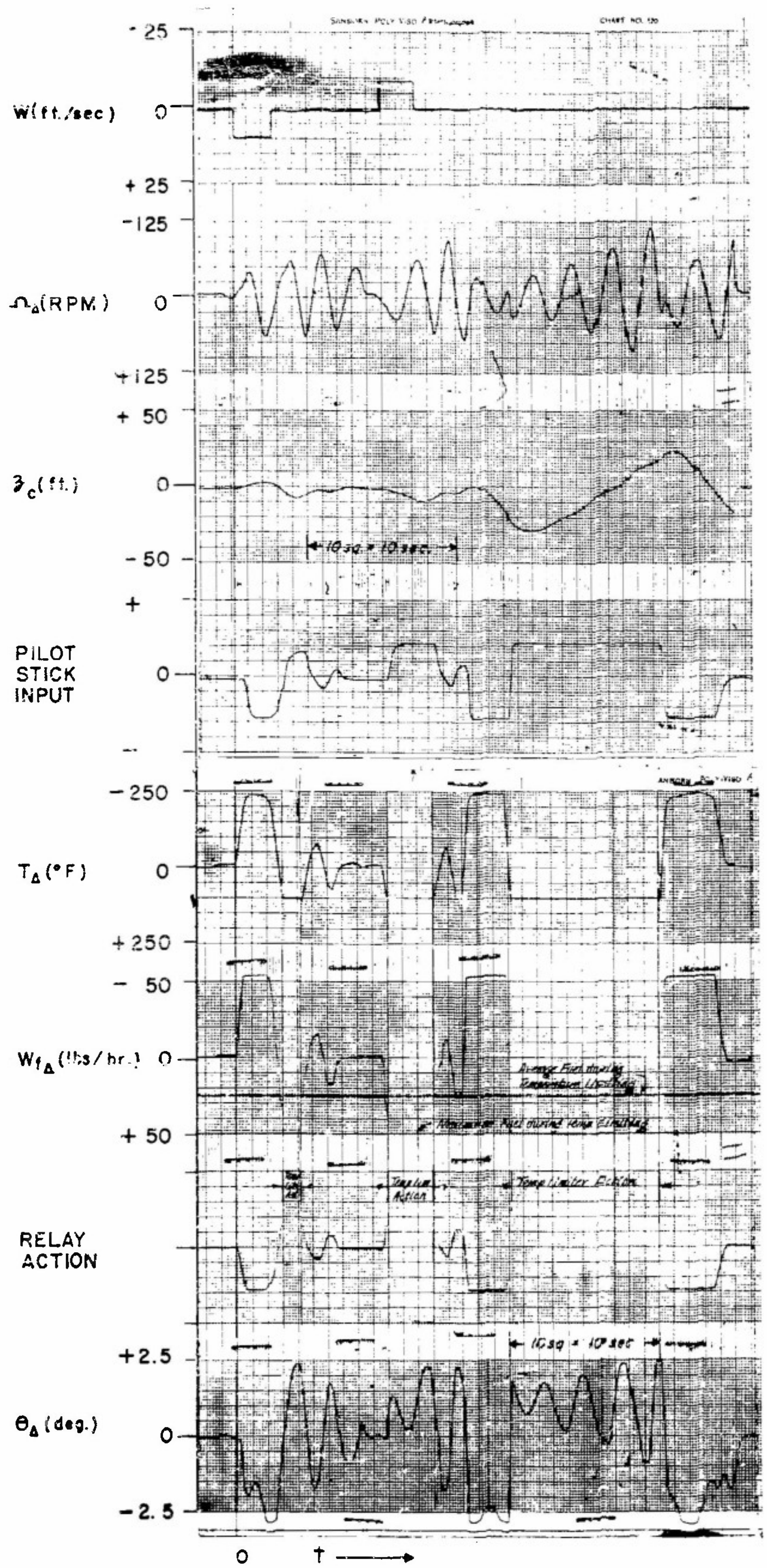

Figure 8.- Run 11, $K_1 = 0$; $K_2 = 4.8$; $K_3 = 0$; $K_4 = 0$

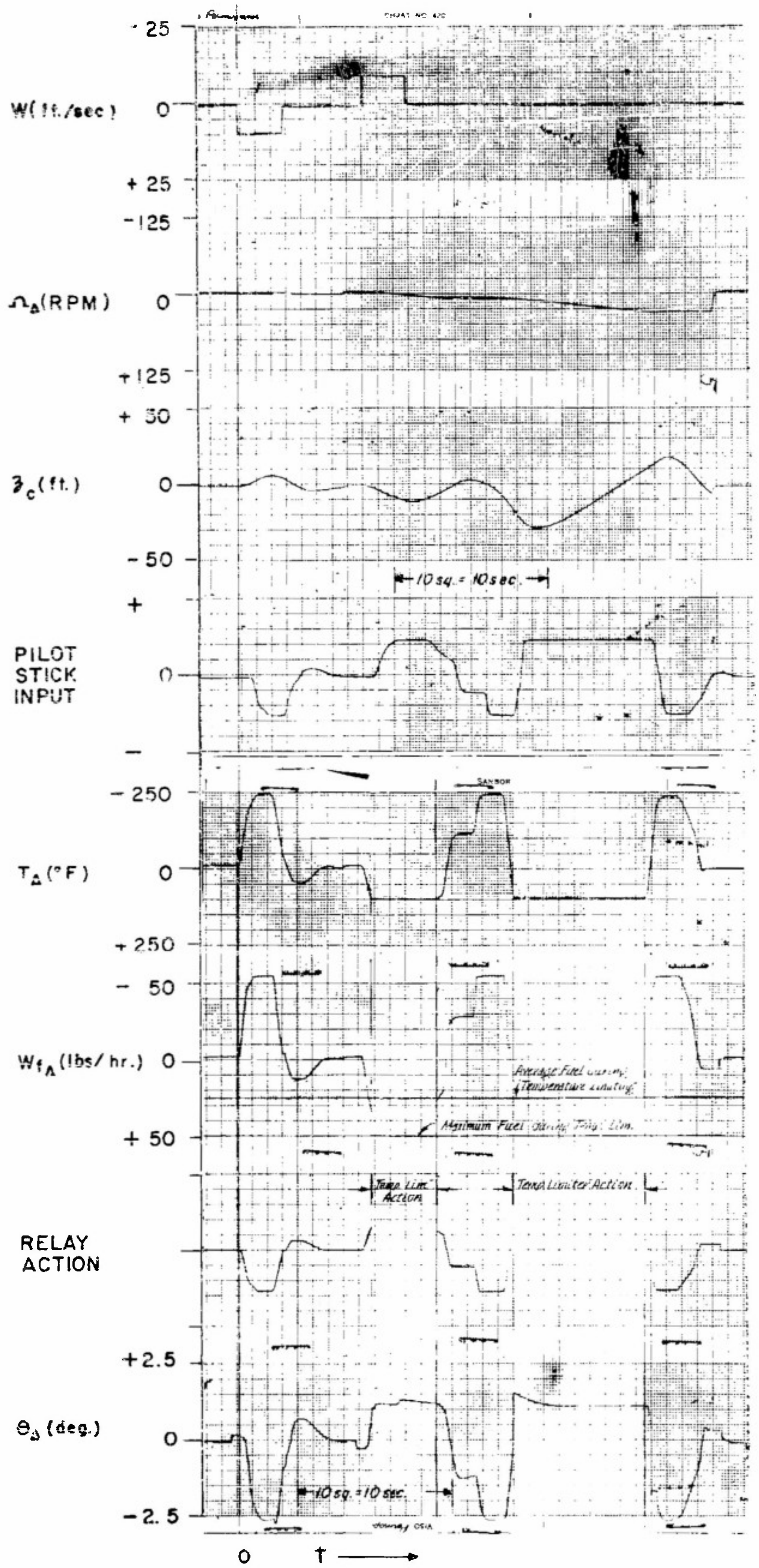

Figure 9.- Run 5, $K_1 = 0$; $K_2 = 0$; $K_3 = 6.0$; $K_4 = 0$

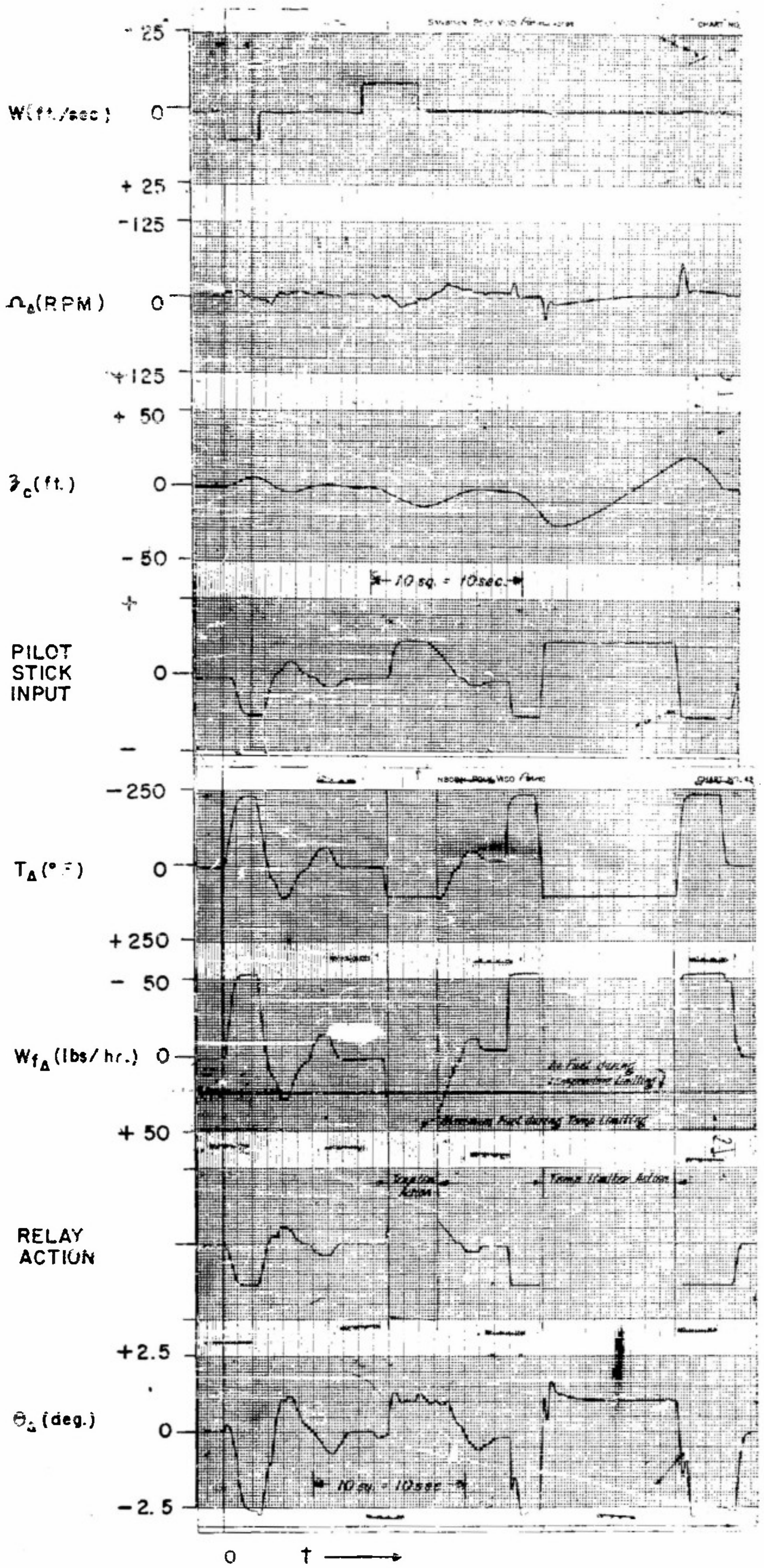

Figure 10.- Run 31, $K_1 = 0.6$; $K_2 = 0.24$; $K_3 = 0$; $K_4 = 0$

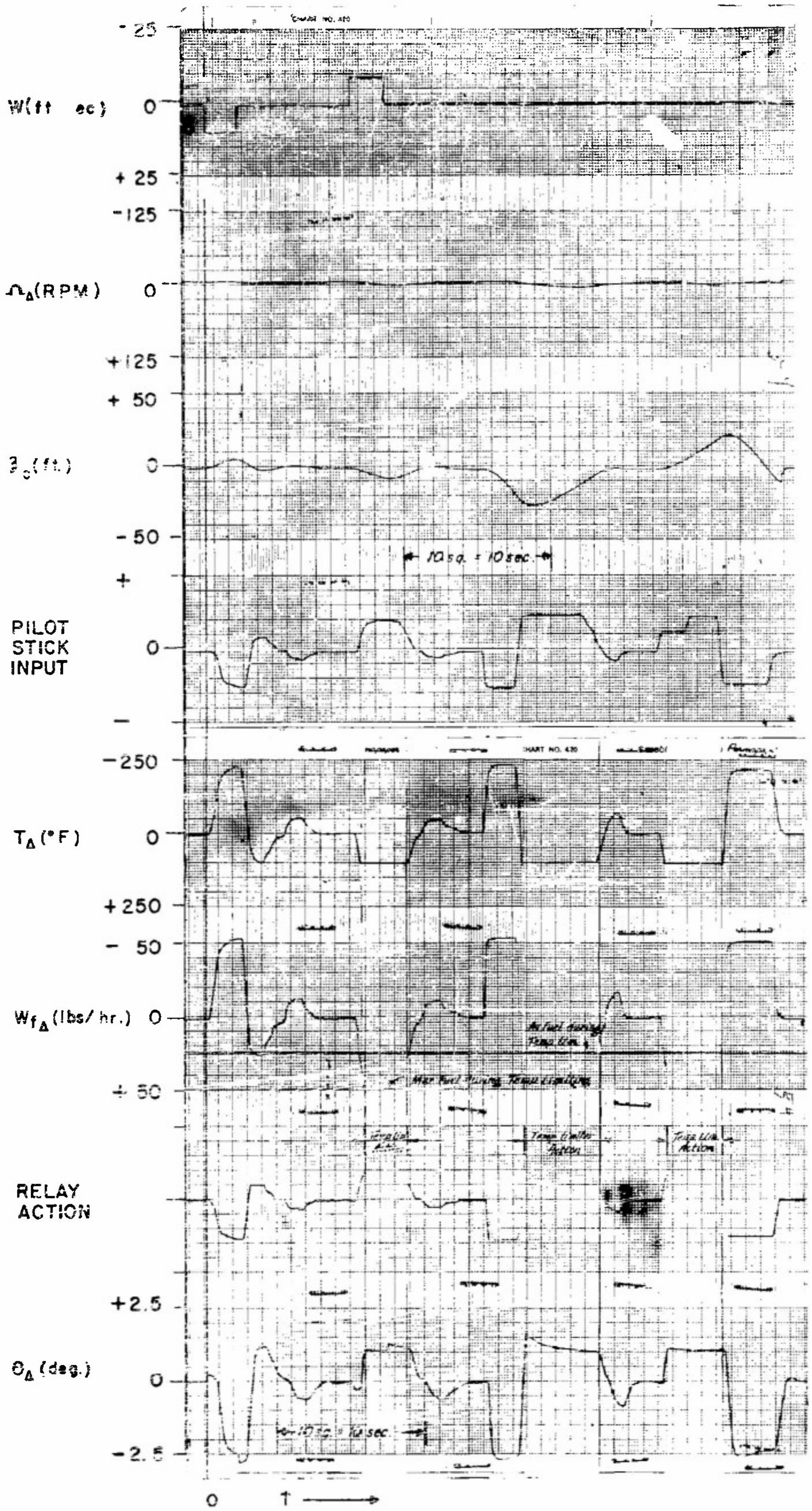

Figure 11.- Run 54, $K_1 = 1.2$; $K_2 = 0$; $K_3 = 3.0$; $K_4 = 0$

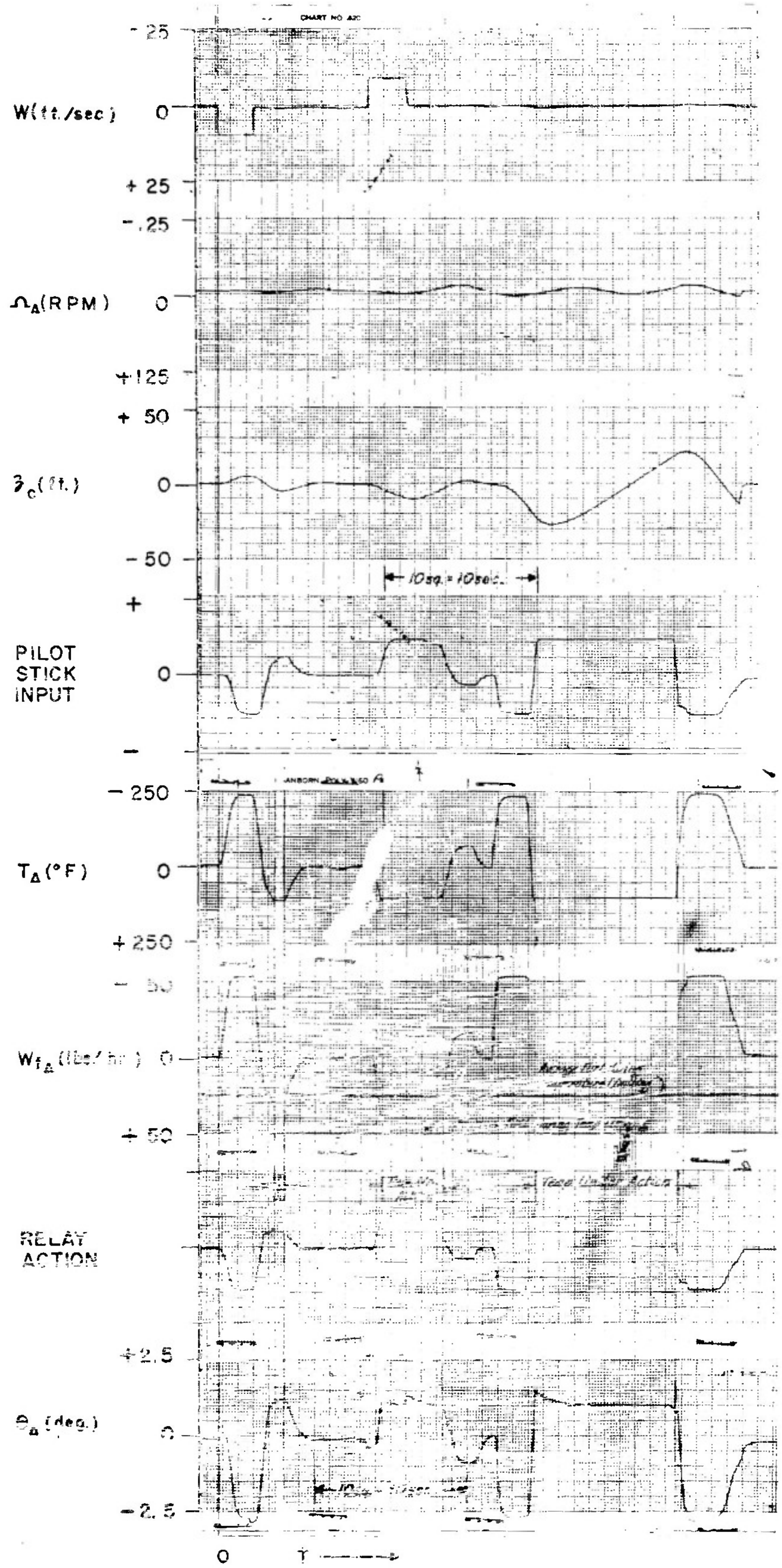

Figure 12 - Run 19, $K_1 = 0$;
$K_2 = 2.4$; $K_3 = 3.0$; $K_4 = 0$

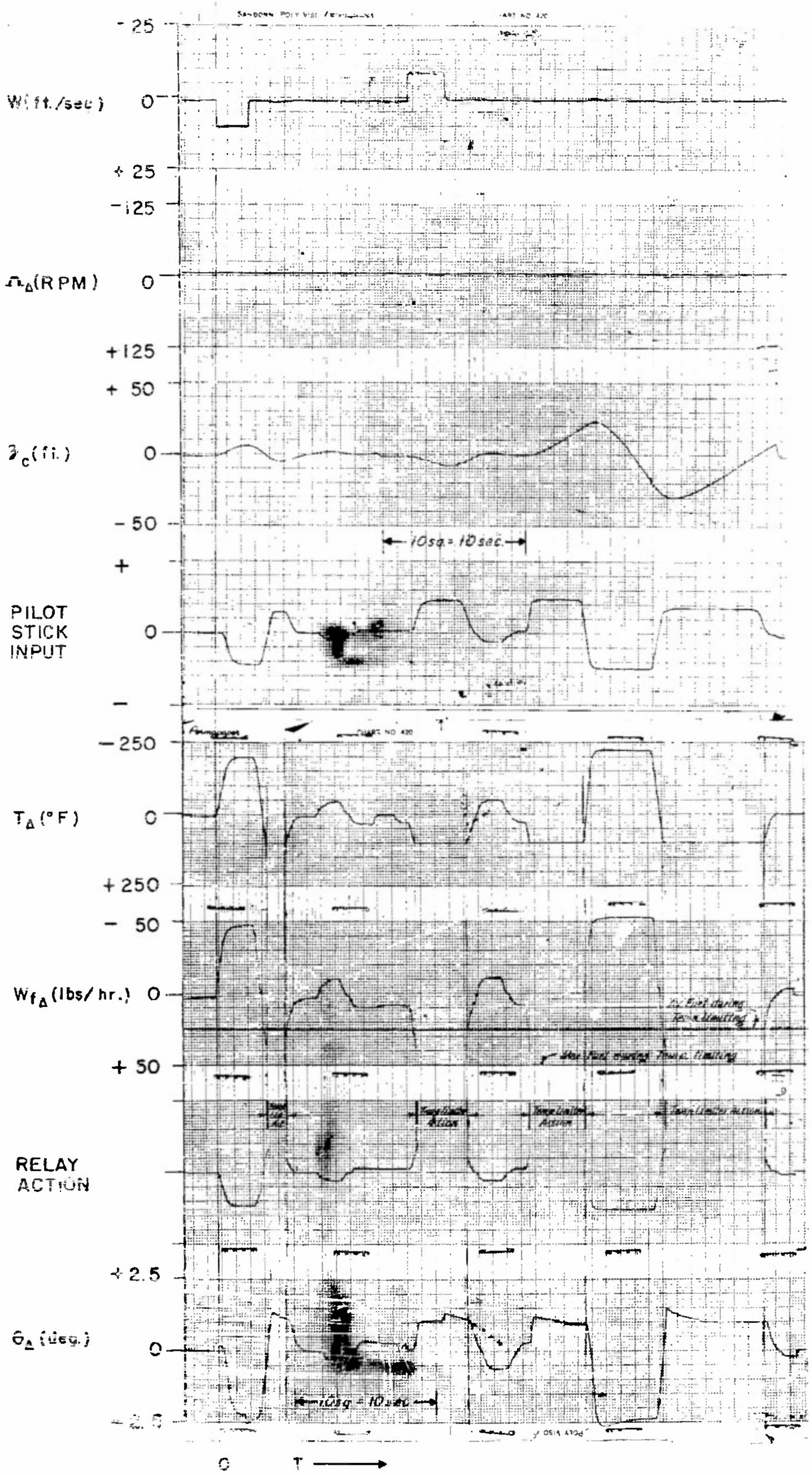

Figure 13 - Run 119, $K_1 = 1.2$; $K_2 = 2.4$; $K_3 = 3.0$; $K_4 = 0$

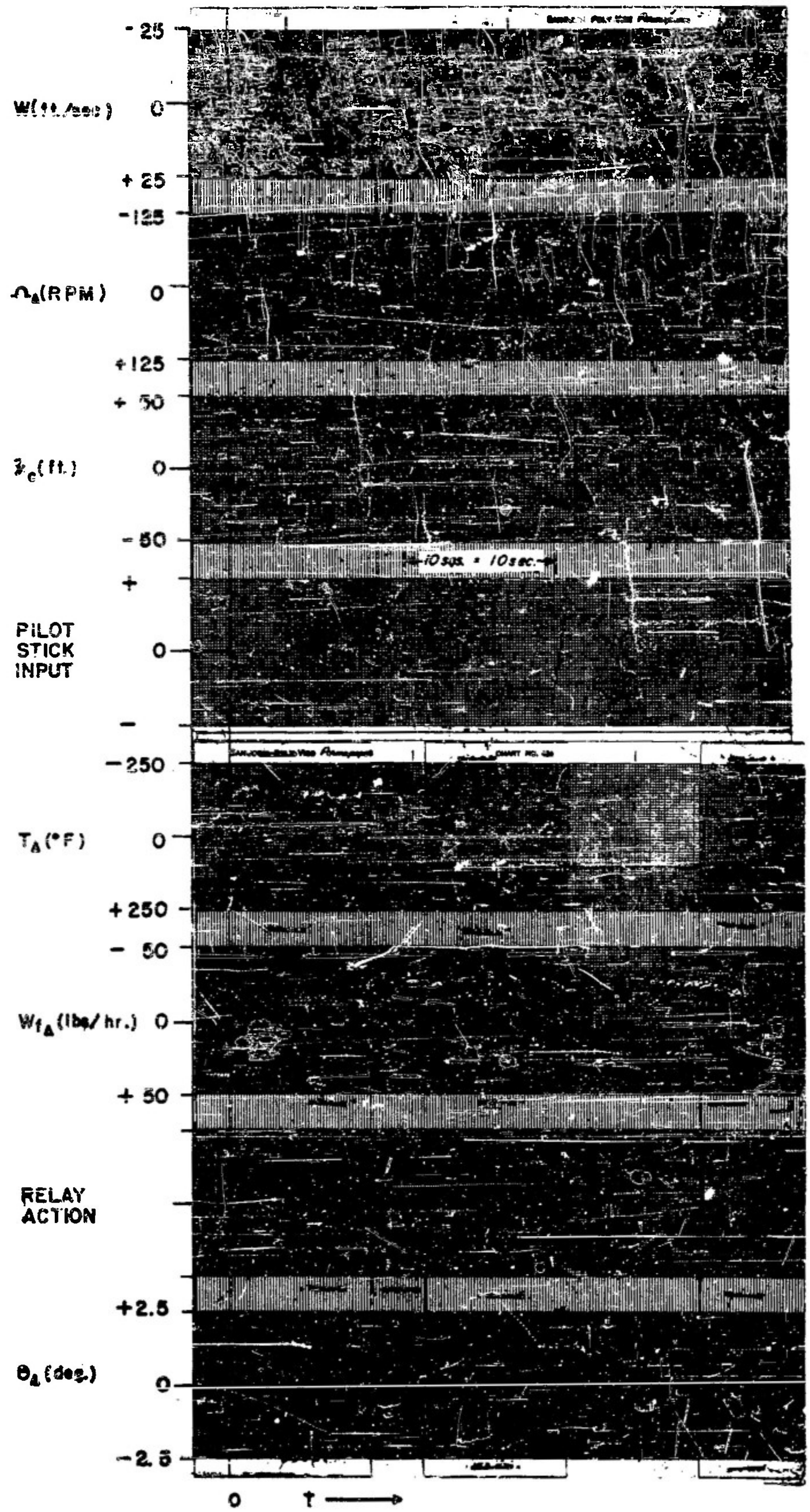

Figure 14. - Run 19-1, $K_1 = 0$;
$K_2 = 2.4$; $K_3 = 3.0$; $K_4 = 0.2$

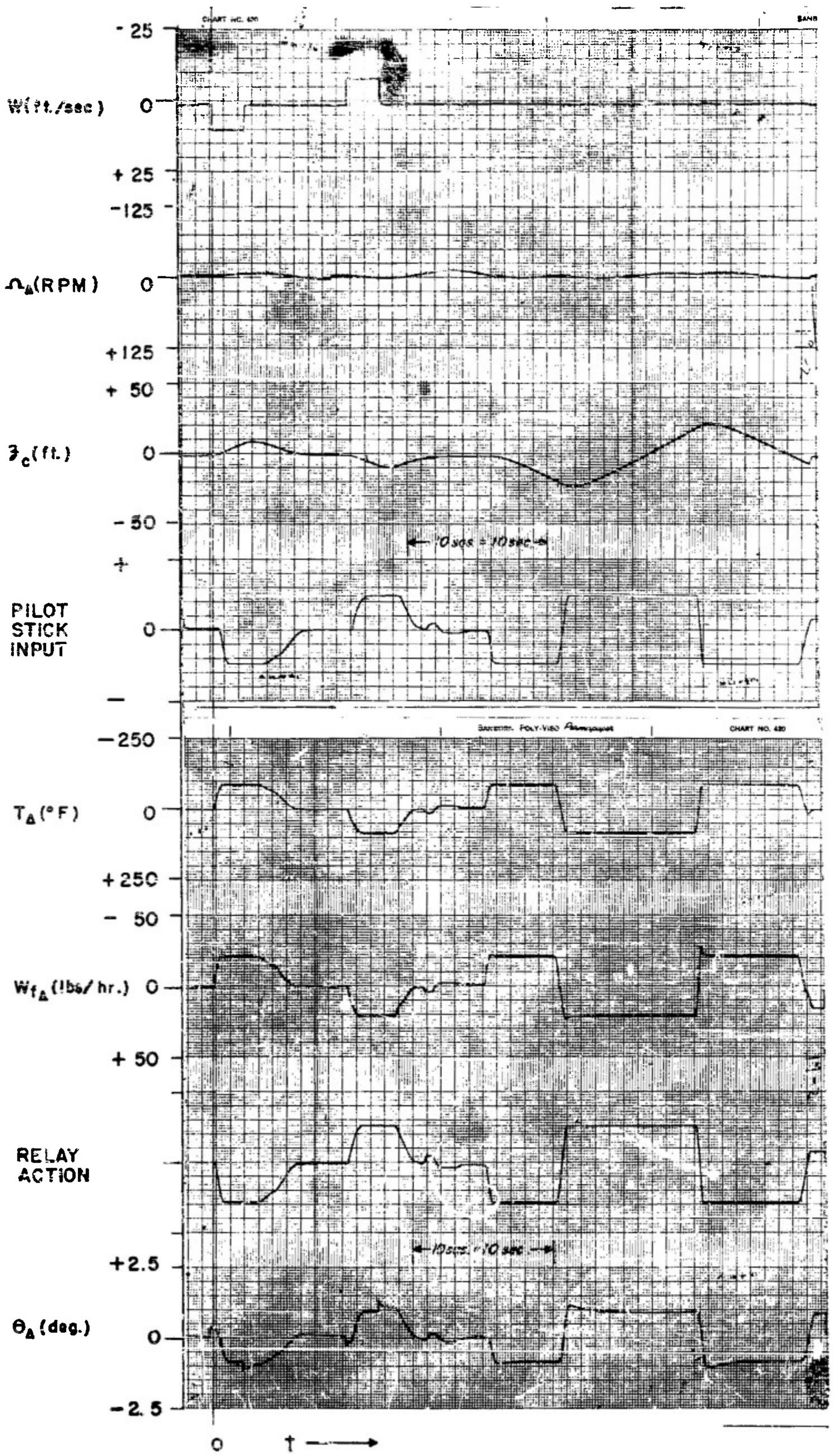

Figure 15. - Run 19-2, $K_1 = 0$; $K_2 = 2.4$; $K_3 = 3.0$; $K_4 = 0.5$

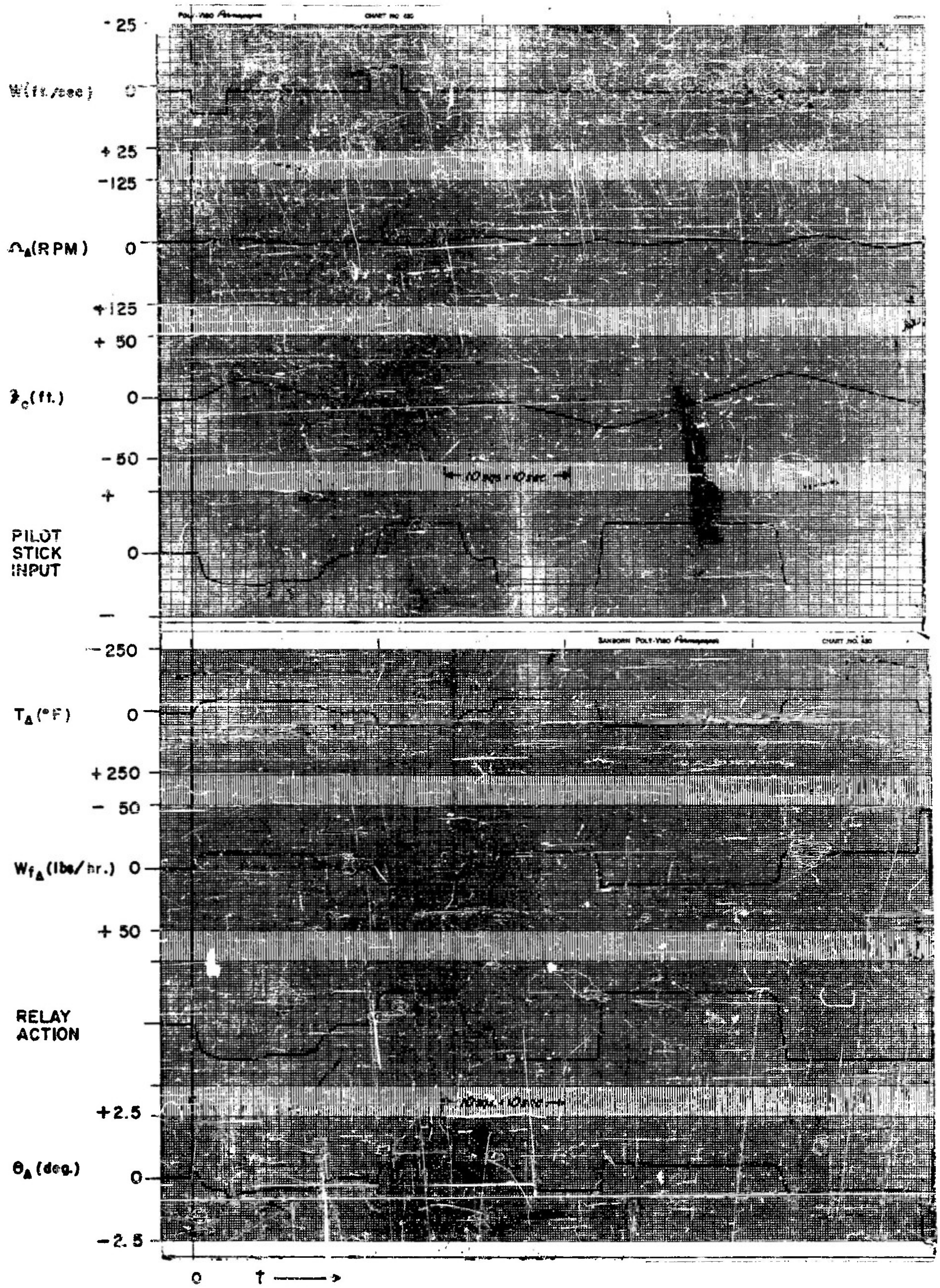

Figure 16. - Run 19-3, $K_1 = 0$; $K_2 = 2.4$; $K_3 = 3.0$; $K_4 = 1.0$

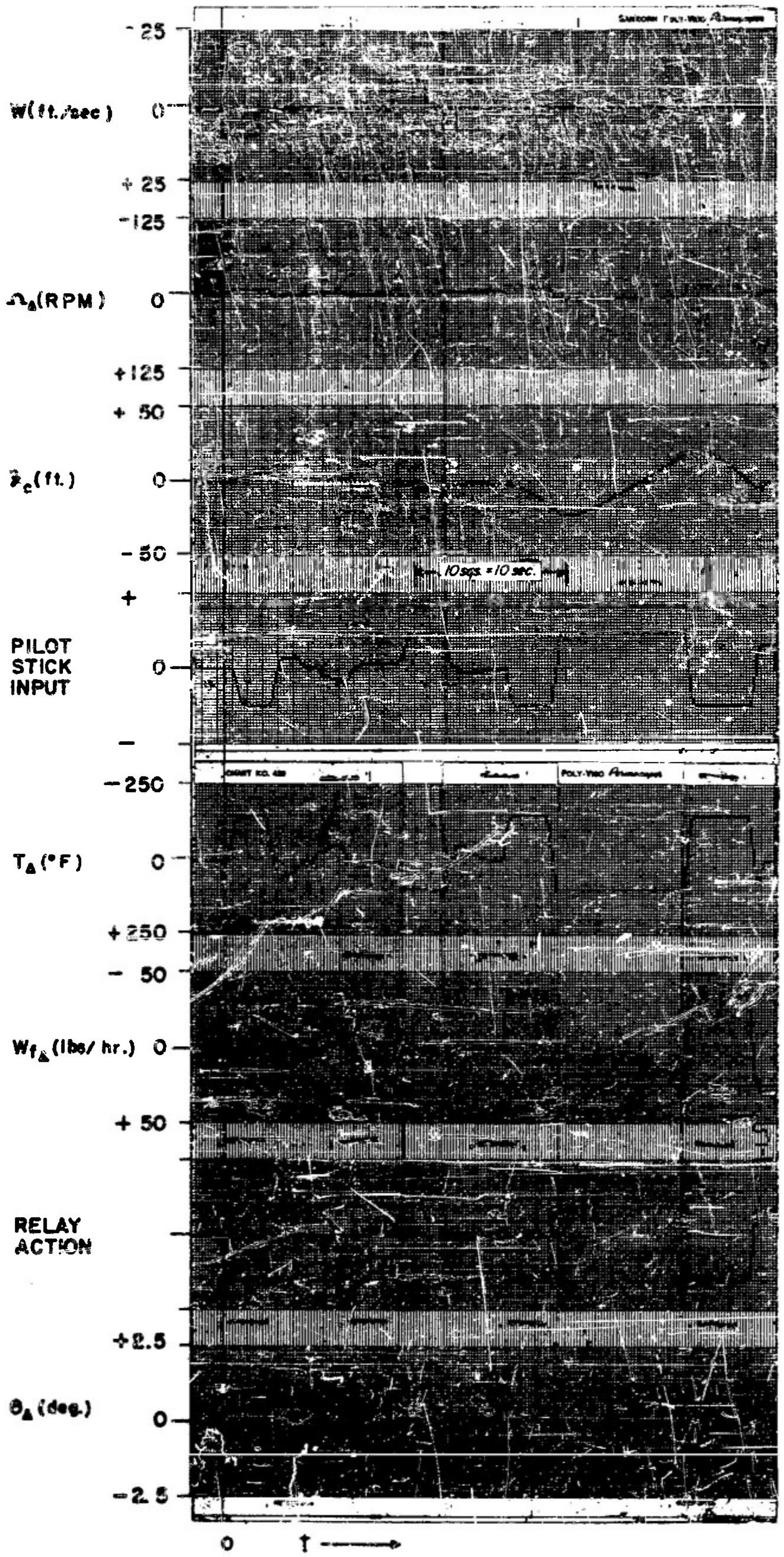

Figure 17. - Run 51-1, $K_1 = 1.2$; $K_2 = 0$; $K_3 = 0$; $K_4 = 0.2$

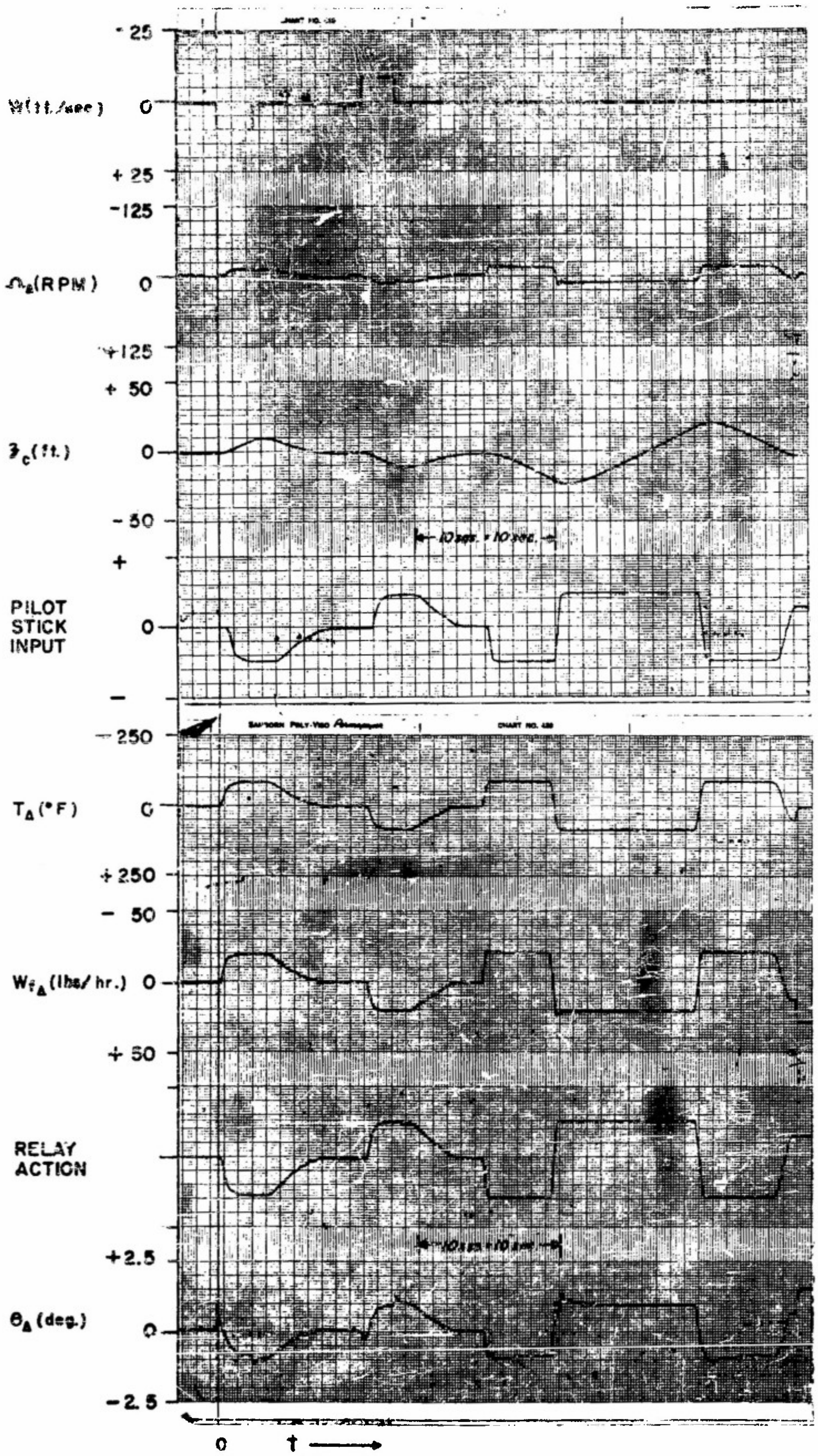

Figure 18.- Run 51-2, $K_1 = 1.2$; $K_2 = 0$; $K_3 = 0$; $K_4 = 0.5$

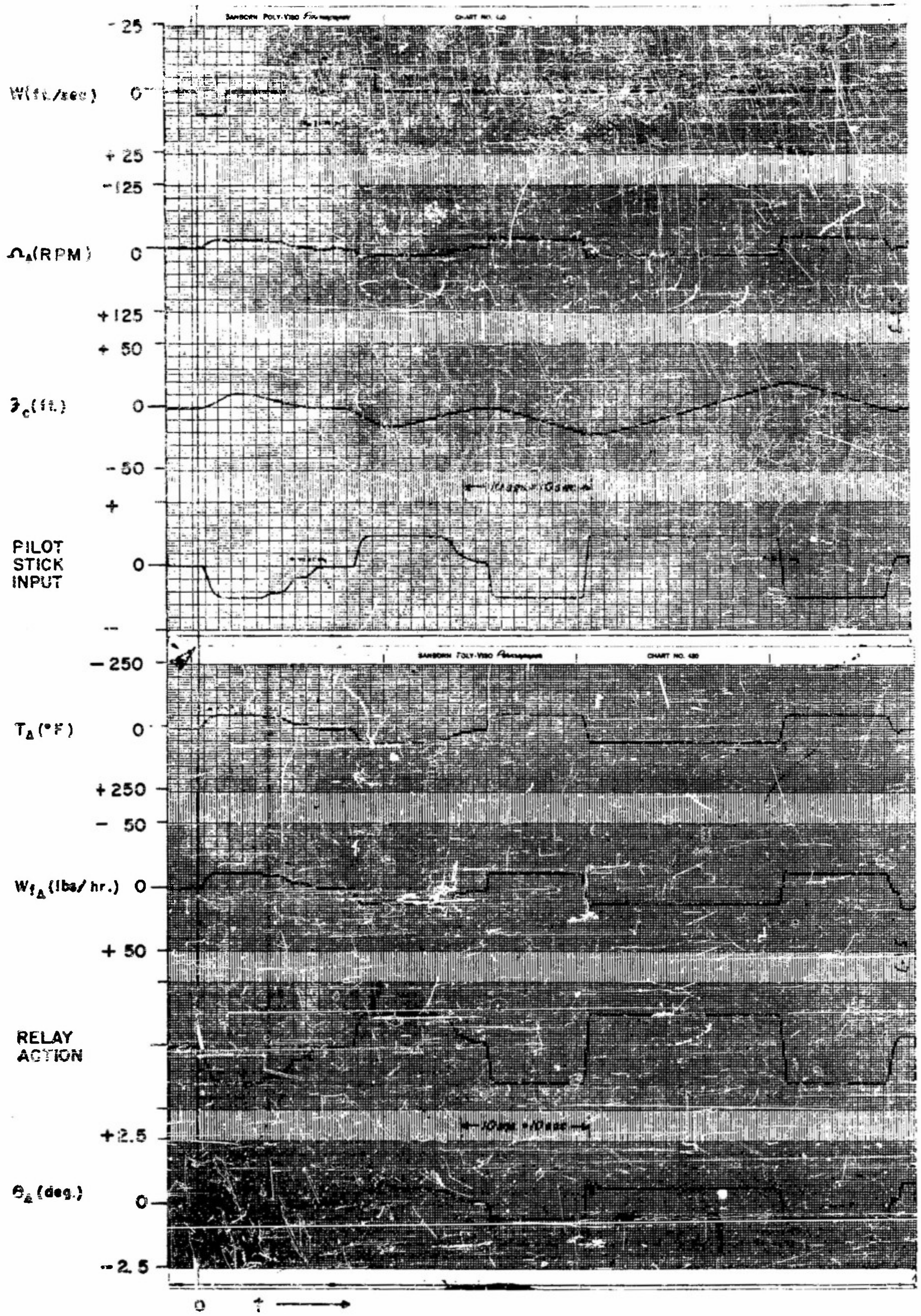

Figure 19.- Run 51-3, $K_1 = 1.2$; $K_2 = 0$; $K_3 = 0$; $K_4 = 0$

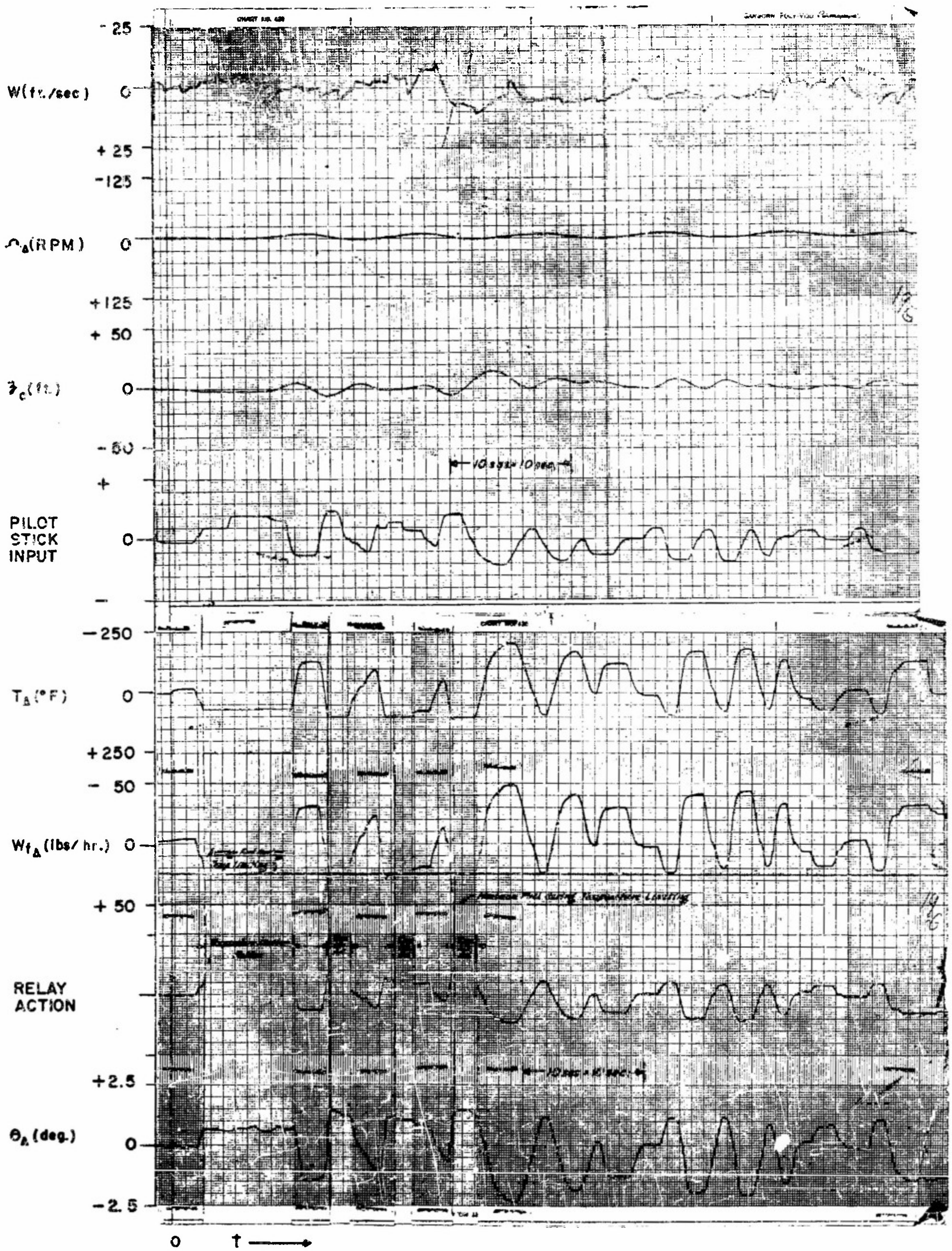

Figure 20. Run 19-G, $K_1 = 0$; $K_2 = 2.4$; $K_3 = 3.0$; $K_4 = 0$

SECTION IV
CONCLUSIONS

The conclusions which have been derived from this study are two-fold in scope. There are first the conclusions concerning the various controls which have been examined, and second, there are the conclusions concerning the logical future course of this work.

It has been stated that a choice between the rpm controls already studied must be based on information on required tightness of control, responsiveness of the helicopter, control complexity, and estimated cost. From the standpoint of minimum complexity the use of a control involving only proportional and integral feedbacks would be advisable; but to gain maximum tightness of control, rate should be added with a consequent increase in complexity, etc. The responsiveness of the helicopter varies only slightly from one optimum control to another, although it definitely degenerates when a considerable degree of temperature control is added. All rpm control configurations studied, except proportional alone or integral alone, exhibited worthwhile performance characteristics.

The present research involving turbine engines was prematurely brought to a close because of the increased interest in piston engines, and for this reason there remain several points of interest yet to be studied. Further work on control types would involve extension of the range of gain settings for the integral plus rate control, and consideration of on-off, lagged-rate and other controls. There is a possible need for compressor surge control and minimum fuel control. Optimalization, in consideration of inputs other than gusts, might be worthwhile as well as consideration of component failures. Some emphasis should be given also to fail safe devices and emergency controls.

SECTION V
APPENDIX A-SYMBOLS

SYMBOLS

Q	torque (ft lbs)
w	gust velocity (ft/sec)
$\dot{\beta}$	flapping velocity (rad/sec)
θ	collective pitch angle (degrees)
Ω	rotational velocity (rad/sec)
z_c	altitude (ft)
S	Laplace operator
t	time
W_f	fuel flow (lbs/hr)
T	turbine inlet temperature (degrees)
K	control gain
N	gear ratio
B	
b	
a	
δ_0	These are defined in Reference (13)
δ_1	
δ_2	
ρ	
m	
$\bar{R}$	
W	gross weight (lbs)
HP	horsepower
I_{tot}	moment of inertia of all rotating parts about shaft axis
I	component moment of inertia

SUBSCRIPTS

A	aerodynamic (refers to rotor)
E	engine
Δ	increment
4, 1	proportional
5, 2	integral
6, 3	rate
S or 0	steady state
w	portion due to gust
$\dot{\beta}$	portion due to flapping velocity
θ_Δ	portion due to change in θ.
Ω_Δ	portion due to change in Ω.
$\dot{z}_c$	portion due to vertical velocity
p	pilot's input

SECTION VI
APPENDIX B-EQUATIONS

The purpose of this portion of the report is to show the derivation of and philosophy behind the equations used to simulate the helicopter engine system.

1. Helicopter

The principal equations expressing the torque, blade flapping and vertical motion of a single-rotor helicopter restricted to vertical flight as derived in reference (13) are:

$$Q_4 = Q_0 + Q_\omega \omega + Q_{\dot{\beta}} \dot{\beta} + Q_{\theta_\Delta} \theta_\Delta + Q_{\Omega_\Delta} \Omega_\Delta + Q_{\dot{z}_c} \dot{z}_c$$

$$0 = M_0 + M_\omega \omega + M_{\dot{\beta}} \dot{\beta} + M_{\theta_\Delta} \theta_\Delta + M_{\Omega_\Delta} \Omega_\Delta + M_{\dot{z}_c} \dot{z}_c + M_\beta \beta + M_{\ddot{\beta}} \ddot{\beta}$$

$$0 = T_0 + T_\omega \omega + T_{\dot{\beta}} \dot{\beta} + T_{\theta_\Delta} \theta_\Delta + T_{\Omega_\Delta} \Omega_\Delta + T_{\dot{z}_c} \dot{z}_c + T_{\ddot{z}_c} \ddot{z}_c + T_{\ddot{\beta}} \ddot{\beta}$$

where all steady-state values are denoted by subscript 0 and the Q, M, and T coefficients are defined in reference (13) as are the quantities, listed below, from which the Q's, M's and T's are computed.

$$B = 0.97 \qquad \delta_1 = -0.0216$$
$$b = 3 \qquad \delta_2 = 0.40$$
$$a = 5.7 \qquad \rho = 0.002378$$
$$\delta_0 = 0.0087 \qquad m = 0.15$$
$$R = 16.5$$

All equations given here or in the DISCUSSION are linearized and have been subjected to a Laplace transformation so that they may be treated algebraically.

The magnitudes of the variables involved in this analysis are based on deviations from steady-state conditions and are consequently zero while the helicopter is hovering in smooth air with no variations in rpm or rotor collective pitch. Thus the set or steady-state operation values such as Q_o, M_o, T_o, θ_o and Ω_o are used only for computational purposes and are neglected in the REAC equations. The steady-state quantities are mainly used in matching helicopter and engine, e.g., the weight (W) of the helicopter was determined as the negative of steady-state thrust (T_o) which in turn was derived from the aerodynamics and steady-state rpm of the rotor. The basic matching between engine and rotor was in the HP output and HP required, respectively. Once the approximate engine HP output(discussed later along with the engine equations) was known, the physical dimensions, steady-state torque, thrust and rpm of the rotor could be quickly computed. Steady-state thrust or helicopter weight was found to be

$$T_o = W = 3125.5 \text{ lbs.}$$

HP required equals HP output, or

$$HP_{req} = \frac{\Omega_o Q_o}{550} = 224 \qquad \text{(See discussion on engine)}$$

Combining this equation with that given in reference (13) expressing Q_o as a function of Ω_o and other variables gives a means of determining both Ω_o and Q_o. θ_o was arbitrarily chosen as 0.1746 radians = 10°. The calculated values of Ω_o and Q_o were 35.77 radians/second and 3444.23 ft lbs, respectively.

The final equations expressing the torque, flapping and vertical motion of the helicopter are:

$$\frac{Q_A}{N} = .915\omega + 13.15\dot{\beta} + 30.2\theta_\Delta + .58\Omega_\Delta - .915\dot{z}_c \qquad (1)$$

$$\begin{aligned} 0 = & -284.1\omega - 3411.7\dot{\beta} + 2132\theta_\Delta + 40.9\Omega_\Delta \\ & + 284.1\dot{z}_c - 287382\beta - 224.6\ddot{\beta} \end{aligned} \qquad (2)$$

$$\begin{aligned} 0 = & \ 79.9\omega + 852.3\dot{\beta} - 532\theta_\Delta - 13.5\Omega_\Delta \\ & - 79.9\dot{z}_c - 97.1\ddot{z}_c \end{aligned} \qquad (3)$$

where N is the ratio between engine and rotor speeds and was chosen as 17.5.

2. Engine

The relationships as given in Reference 16 expressing the dynamics of a single-spool turbo-prop engine are $Q_E = Q_E(W_f, \Omega)$ and $T = T(W_f, \Omega)$ which, when expanded as total derivatives, assuming linearity, become:

$$Q_E = \left.\frac{\partial Q_E}{\partial W_f}\right|_{\Omega} W_{f_\Delta} + \left.\frac{\partial Q_E}{\partial \Omega}\right|_{W_f} \Omega_o$$

AND

$$T_\Delta = \left.\frac{\partial T}{\partial W_f}\right|_{\Omega} W_{f_\Delta} + \left.\frac{\partial T}{\partial \Omega}\right|_{W_f} \Omega_o$$

The partial derivatives $\left.\frac{\partial Q_E}{\partial W_f}\right|_{\Omega}$, $\left.\frac{\partial Q_E}{\partial \Omega}\right|_{W_f}$, $\left.\frac{\partial T}{\partial W_f}\right|_{\Omega}$, and $\left.\frac{\partial T}{\partial \Omega}\right|_{W_f}$ were obtained from the performance curves given in reference (17) which is a model specification for the Continental Turbomeca shaft turbine Model "Artouste-I". The performance curves relate shaft HP to output shaft rpm for various values of the parameters specific fuel consumption and turbine inlet temperature. It was from slopes of these curves that the partial derivatives were obtained.

Because the chosen steady-state operating point of the engine determined to a large extent the required size and characteristics of the rotor, a short discussion on the subject is included here. From considerations given in helicopter reports to the relationship between required takeoff and required hovering HP it was decided that the present helicopter would hover at 80 percent of maximum takeoff HP. This resulted in a steady-state engine shaft HP of 224 since the maximum takeoff is 280 HP. The engine output shaft rpm is maintained at 6000 during all operating conditions, thus the engine operating point is determined. Further consideration must be given to the operating point because of the existence of the maximum temperature limiting required to prevent damage to the turbine. The steady-state operating point for hovering will vary with factors such as gross weight, and consequently, even though the operating curves slopes vary little in passing from one operating condition to another, the response of the helicopter to increased power demands will not be the same when operating considerably below the maximum inlet turbine temperature line as when operating close to it. This situation would normally require the analysis to be run at several operating points along the 6000 shaft rpm output line with temperature limiting existing at some predetermined value (in this case, 1472° F). Since factors such

as load must change to require new operating points in hovering it can be seen that for each case a set of steady-state conditions for both helicopter and engine must be computed. It was felt that, since the controls investigated were not intended to be tailored for any specific helicopter or engine, variations in engine operating curves with HP output were of little interest from the present standpoint. The effect of changes in helicopter load would be taken into account approximately by varying the amount the temperature could increase above steady-state during transient power increases. The minimum allowable temperature increase (above steady-state) considered was 50° while the temperature at the actual steady-state point is 1305° F and consequently temperature limiting normally occurs at 157° F above steady-state. During the greater portion of the analysis the operating point was considered to be 100° F below the maximum allowable turbine inlet temperature; or at 1362° F. The fuel flow at the steady-state operating point (T_e = 1305° F, Ω_s = 6000 rpm) is 50 lbs/hr below that fuel flow corresponding to the allowable temperature and therefore the maximum fuel flow input change allowed the pilot for full stick movement in either direction was somewhat arbitrarily chosen as 50 lb/hr.

The partial derivatives in the engine torque and temperature equations were computed at the chosen operating point and are given in the equations:

$$Q_E = 1.45 W_{f_\Delta} - .649 \Omega_\Delta$$

AND

$$T_\Delta = 3.87 W_{f_\Delta} - 1.72 \Omega_\Delta$$

There is a lag between the time the fuel valve setting is changed and the time that fuel energy is actually available to cause a change in engine torque or turbine temperature. From NACA engine transient data (Reference 20) it can be seen that this lag is approximately first order with 0.15 second time constant and its inclusions in the equations results in:

$$Q_E = \frac{1.45}{1+.15S} W_{f_\Delta} - .649 \Omega_\Delta \qquad (4)$$

AND

$$T_\Delta = \frac{3.87}{1+.15S} W_{f_\Delta} - 1.72 \Omega_\Delta \qquad (5)$$

Nine variables, six of which are dependent, are involved in the equations 1 to 5 inclusive. Since 6 dependent variables require 6 simultaneous equations to give a solution, an additional equation must be written; that is the expression relating the unbalance in torque between rotor and engine to the angular acceleration of this system, thus:

$$Q_E - Q_A = I_{tot}\,\dot{\Omega}$$

where I_{tot} is the total moment of inertia normally including engine, rotor, gear box, tail rotor, and fly wheel; but in this case only rotor and engine were included because the moment of inertia of the rest of the system was considered to be rather small as compared with that of the rotor. Because the rotational speeds of rotor and engine differ by a factor $N = 17.5$, the unbalanced torque equation must be adjusted slightly. $Q_E = I_E \dot{\Omega}_E$ and $Q_A = -I_A \dot{\Omega}_A$ where $\Omega_E = N \Omega_A$. Therefore $Q_A = -I_A \frac{\dot{\Omega}_E}{N}$. For steady-state operation ($\dot{\Omega}_E = 0$) the unbalance in torque must be zero, this statement being expressed as $Q_E - \frac{Q_A}{N} = 0$. We may then write:

$$Q_E - \frac{Q_A}{N} = \left(I_E + \frac{I_A}{N^2}\right)\dot{\Omega}_E \qquad (6)$$

which becomes:

$$Q_E - \frac{Q_A}{N} = 2.22\,\dot{\Omega}_o$$

after substituting $I_A = 3(\frac{1}{3} mR^5) = 2.2 N^2$ and $I_E = .71\#ft^2 = .025 slug\,ft^2$ (given) where m is specific mass of rotor blade and R is the radius.

Final Equations

Because equations 1-6 inclusive have been subjected to a Laplace transformation they may be treated as algebraic equations. Equations 1, 4, and 6 were combined forming the resultant:

$$\begin{aligned}\ddot{\Omega}_\Delta = {} & 4.35\,W_{f_\Delta} - 3.69\,\Omega_\Delta - 2.75\,w - 39.4\,\dot{\beta} - 90.1\,\theta_\Delta \\ & + 2.75\,\dot{z}_c - 7.20\,\dot{\Omega}_\Delta - .411\,\dot{w} - 5.91\,\ddot{\beta} - 13.5\,\dot{\theta}_\Delta \\ & + .411\,\ddot{z}_c \end{aligned} \qquad (9)$$

Equations 2, 3 and 5 are re-written in a form more adaptable to the REAC as:

$$\ddot{\beta} = -1.265\omega - 15.18\dot{\beta} + 9.50\,\theta_\Delta + .182\,\Omega_\Delta + 1.265\,\dot{\mathfrak{z}}_c - 1278\,\beta \qquad (2a)$$

$$\ddot{\mathfrak{z}}_c = .823\,\omega + 8.77\dot{\beta} - 5.48\,\theta_\Delta - .139\,\Omega_\Delta - .823\,\dot{\mathfrak{z}}_c + .631\,\ddot{\beta} \qquad (3a)$$

and $$\dot{T}_\Delta = 25.8\,W_{f_\Delta} - 11.48\,\Omega_\Delta - 6.67\,T_\Delta - 1.72\,\dot{\Omega}_c \qquad (5a)$$

The above equations plus the two control equations given in the text constitute the basis for the REAC simulation.

SECTION VII
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